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Inequality aversion to posterity

Parouty, Mehraj Bin Yasaar

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Inequality Aversion to Posterity: Discounting Human Lives



Inequality Aversion to Posterity: Discounting Human Lives

M.B.Y. Parouty

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Mehraj Bin Yasaar Parouty

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Promotor:	Prof. dr. M.J. Postma
Beoordelingscommissie:	Prof. dr. B.W. Lensink
	Prof. dr. R. Sanderman
	Prof. dr. A. Salhi
Paranimfen:	Rika(H) A. Katerborg-van Dijk
	Paul G. Journée

Contents

1	General Introduction	7
I	The Need for Differential Discounting	13
2	Ethical Debates and Proposed Resolutions	15
2.1	Health investments	17
2.1.1	Measures of health	17
2.1.2	Ethical CEAs	19
2.2	An extreme example	21
2.2.1	Overview of approaches	21
2.2.2	Results	25
2.2.3	Discussion	28
2.3	Motivation matters	29
2.a	Appendix: Pathogenesis cervical cancer (Stanley (2010))	31
3	The Social Discount Rate	39
3.1	The Ramsey discount rate	40
3.1.1	Ramsey brief	40
3.2	Ramsey rate for health?	43
3.2.1	The marginal substitution between health and wealth	44
3.3	A differential rate for health	45
3.3.1	Components of the Ramsey rate	46
3.4	Discussion	50

4	Questionnaire Development	57
4.1	Problem framing	58
4.1.1	Question formulation	59
4.1.2	Response types	59
4.2	Rule rationality	60
4.3	Preliminary data analysis	62
4.a	Appendix: Final questionnaire	63
5	Empirical Evidence	77
5.1	Methods	78
5.2	Results	81
5.3	Discussion	88
II	A Framework for Differential Discounting	95
6	Differential Discounting	97
6.1	Discounting dynamism	99
6.2	Dynamics denotation	101
6.3	Denouement	103
7	A Relaxation of Invariance	109
7.1	A covariance representation of coordinate transformations	111
7.2	A derivation with commodities	115
7.3	An illustration in welfare	120
7.3.1	An application in health economic evaluations	123
7.4	Discussion	127
7.a	Appendix: Sraffa's production for subsistence approach to health	129
8	Probability Measures	135
8.1	Distribution derivation	137
8.1.1	Probability assignment	139
8.1.2	Differential equation	141
8.1.3	Distribution	142

8.1.4	Remarks	144
8.2	Probability of the future monetary value of health . .	146
8.3	Discussion	149
8.a	Appendix: The Poisson process	151
9	An Aspiration Avenue	157
9.1	Utility and aspiration	159
9.1.1	The Probabulity concept	160
9.1.2	The Aspiration utility	161
9.2	Temporal aspiration	162
9.2.1	The marginal monetary value of health . . .	164
9.3	Discussion	167
9.a	Appendix: An illustration on discounted life-expectancy	169
10	General Discussion	177

List of Figures

3.1	Graph from Ramsey’s ” <i>Mathematical theory of saving</i> ”	46
5.1	Annual discount rate for money(●) and health(◇) plotted against time of delay.	82
5.2	Annual discount rate for money plotted against time of delay. Female(◁) respondents are compared to male(◇) respondents.	83
5.3	Annual discount rate for money plotted against time of delay. Respondents who are optimistic (○) are compared to pessimistic (★) respondents.	84
5.4	Annual discount rate for health plotted against time of delay. Respondents belonging to the upper 50% regarding health status (▷) are compared to those belonging to the lower 50% regarding health status (*).	85
10.1	Space-Time discounting	181

List of Tables

- 2.1 Country specific discount rates for costs and health . 20
- 2.2 Step-wise discounting approaches proposed in literature 24
- 2.3 Overview of different discounting approaches introduced above 26
- 2.4 Discounted health outcomes of HPV vaccination . . 27

- 5.1 Partitioning of overall sample set in mutually exclusive subsets 81
- 5.2 Linear regression fits on the discount rate for money ($r = \alpha + \beta t$) and 95% confidence intervals, *C.I.*'s . . 86
- 5.3 Linear regression fits on the discount rate for quality of life ($r = \alpha + \beta t$) and 95% confidence intervals, *C.I.*'s 86

Chapter 1

General Introduction

Animals, including human beings, generally prefer to consume now than to consume in the future. Primates, such as *Saguinus oedipus*, for example, are known to act as though they discount¹ future consumption (Rosati *et al*, 2006). Among the human species, the concept of interest rate, for example, is common, interestingly, even among children. A hundred euro note in a bank account today is generally accepted to be more valuable than the same euro note in, say, one year's time. Consideration of such a characteristic by economists began early in the history of their discipline, with scottish economist John Rae's *Sociological theory of capital* in 1834.

Rae (1834) first set the scene by assuming that decisions were guided by pure reason, without any emotional and/or psychological motives, such as impatience or diminishing marginal utility². With that foundation, he then sequentially considered the psychological motives as they became apparent. Such economic foundations for intertemporal choice however lost consensus when Paul Samuelson (1937) introduced the discounted-utility (*DU*) model with a 5-page

¹A future consumption is *devalued* compared to a current one.

²Utility is a measure of pleasure, derived from consuming. Diminishing marginal utility means that the pleasure derived from consuming an extra grape, say, decreases as we consume more grapes.

article in which he condensed the many conflicting psychological motives into a single parameter, the discount rate. Unanimity of the model was immediate and prevails until today. Needless to say that the axiomatic derivation of the *DU* model has been numerous (Fishburn, 1970, Koopmans, 1960, Lancaster, 1963, Meyer, 1976, Rubinstein, 2003).

An important assumption of the discounted utility model is that the discount rate is positive and invariant across time and across all forms of consumption. The assumption of a rate that is invariant across different types of consumption is central in time preferences; otherwise we would have a certain time preference for, say, kiwis and a different time preference for grapes, say. And the assumption of a rate that is invariant across time implies that plans are consistent; otherwise the plans that we make today will constantly be changing as we move into the future. Although Samuelson (1937) was skeptic about the validity of the *DU* model, its prevalence is largely due to its simplistic nature and wide range of applicability (Frederick *et al*, 2002).

The beginning of the 80's has seen the inclusion of such a discount rate for human lives. It is well known, however, that the practice of discounting human lives, assuming invariance, impacts heavily on cost-effectiveness analyses of different types of projects. In 1991, for example, the asbestos regulation issued by the Environmental Protection Agency (EPA) was overruled by the U.S. Fifth Circuit Court of Appeals [See 947 F.2d (5th Cir. 1991)] where the judges wrote: "*Because the EPA must discount costs to perform its evaluations properly, the EPA should also discount benefits to preserve an apples-to-apples comparison, even if this entails discounting benefits of a non-monetary nature*".

With regards to the use of a single and only social discount rate, *SDR*, such decisions have spurred much debates over the last few decades. European bodies such as the CvZ ("College voor Zorgverzekerin-

gen”) in the Netherlands, NICE (National Institute of Clinical Excellence) in the UK, SMC (Scottish Medicines Consortium) in Scotland, TLV(“Ta ndvards-och LakemedelsformansVerket”) in Sweden and KCE (“Kennis Centrum”) in Belgium – just to arbitrarily name some – all design, discuss and revise the equitability of their guidelines with regards to discounting human lives (CvZ , 2013, ISPOR, 2013, KCE , 2013). However, with majority of guidelines prescribing a single *SDR*, the bias for cures to be cost-effective compared to preventions is not minimised. In an attempt to bring guidelines in phase with ethics, this thesis is two-fold.

First, I argue that the government should protect our rights as well as the rights of our children, although possibly unborn yet, and the rights of our grandchildren and so on. To that end, policies ought to be drafted so as to include methodologies for valuation of medical interventions that are, as far as possible, unbiased with regards to cures or preventions; while keeping in mind, of course, that valuations are typically carried out in the present time. Although there have been several attempts towards *fair* cost-effectiveness analyses, the lack of their normative validity has resulted in a lack of consensus. I provide a theoretical argument for differential discounting of health outcomes and back up the argument with empirical results from a representative sample of the Dutch population.

Second, I propose to extend current economic theory to allow for differential discounting. I pave way for an economic system that relaxes the invariance assumption by considering an n -commodity economy where each commodity has a specific discount rate function. I formulate a model-consistent expectation by borrowing from a concept of Einstein’s general theory of relativity. Next, I add probability measures to each constituent of the system, based on current psychological theory, and discuss the marriage of the deterministic system coupled with a probabilistic measure for each of its constituents. With such a framework, other possible implementations are valuing life improving and life saving interventions dif-

ferently by implementing differential discounting between life years and quality of life.

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Part I

**The Need for Differential
Discounting**

Chapter 2

Ethical Debates and Proposed Resolutions

¹ Priority setting is necessary for efficient resource allocation. The latter, however efficient it be, should nonetheless be within ethical norms. The 60's and 70's have seen a lot of supply constraints by budget allocators. Health Statistics of the European Commission also noted that hospital bed supplies have reduced by over one million over the past 20 years in EU25 (<http://epp.eurostat.ec.europa.eu/tgm/table.do?tab=table&init=1&plugin=0&language=en&pcode=tps00046>). Certainly, it is not feasible to provide all possible health-care interventions that can conceivably be of some benefit. Hence, recent attempts to aid the efficient allocation of resources have been considerations of cost-effectiveness analyses (CEAs) and broader full-blown health technology assessments (HTAs). However, policies prescribed for discounting of health are known to narrow the choices of medical interventions with an unequal hand.

¹Based on

WESTRA TA, PAROUTY MBY, BROUWER WB, BEUTELS PH, ROGOZA RM, ROZENBAUM MH, DAEMEN T, WILSCHUT JC, BOERSMA C AND POSTMA MJ. (2012) *On discounting health gains from Human Papilloma Virus Vaccination* Value in Health, 15, pg 562-567

PAROUTY MBY, BOERSMA C AND POSTMA MJ (2013) *Discounting human lives: A critique* submitted

Majority of bodies prescribe a discount rate for health equalling that of costs; NICE (2008), for example suggests a discount rate of 3.5% for both costs and health outcomes. However, regardless of the type of medical intervention, costs are generally borne immediately but occurrences of health gains extend through broader time horizons. Moreover, interventions having immediate health gains are direct competitors with interventions that have future health gains. Keeping everything else constant, such as thresholds for cost-effectiveness, the impact of discounting health outcomes is greater for programmes that have different time profiles for costs and health effects, such as vaccines.

Examples of the impact of discounting on cost-effectiveness ratios are numerous. A recent study on the cost-effectiveness of a hypothetical *Helicobacter pylori* vaccine has shown that when using the 1999 Dutch guidelines with a discount rate of 4% for health effects, the vaccine is deemed cost-ineffective while the current discount rate of 1.5% favours the cost-effectiveness of the vaccine a lot more (De Vries *et al*, 2009). Hence, there is, on average, a general cost-effectiveness bias for cures compared to preventions (Brouwer and van Exel, 2004, Keeler and Cretin, 1983, Smith and Gravelle, 2001). This could simply be restated as “cure is better than prevention”, reminding us of a quote from Karl Popper (Miller, 1985) that “in the realm of errors, cure is better than prevention”.

The choice of discount rates affects not only the Net Present Value (NPV) of an intervention but also, since there are many such feasible interventions, affects a whole portfolio of interventions; especially the optimal portfolio of health interventions. And this portfolio, in turn, affects the optimal allocation of available budgets by decision makers who will indefinitely have a huge bundle of curative interventions in the allocation problems. Since the degree of attempt to avert future illnesses seems to indicate that future costs of curative interventions, in turn, will never fall, I wonder whether the assump-

tion of equal discount rates can simply be restated as being inside “the realm of errors”.

2.1 Health investments

Health economists generally view health economic evaluations as *health investments*. A health technology assessment typically involves a comparison of competing programmes (Drummond *et al* , 2005, Gold *et al*, 1996, Simoens , 2009). By comparing two or more programmes, differences in costs and health outcomes can be estimated. This comparison can be summarized in a, so-called, incremental cost-effectiveness ratio (ICER). This ICER is calculated by dividing the estimated difference in costs by the estimated difference in health outcomes. By judging this ICER to a relevant threshold, decision-makers can subsequently be informed regarding the desirability of funding a certain intervention. Guidelines of various countries, amongst which the Netherlands (CvZ , 2013), specify how such evaluations should be carried out and what discount rates to use.

2.1.1 Measures of health

The types of health investments fall into three main categories: a long term health investment such as different types of vaccination programs, a short term health investment such as cancer interventions and most curative interventions and finally, a continuous investment such as an annual screening programme for type-2 diabetes (Bos *et al*, 2005). Health gains provide the justifications towards considering the undertaking of an intervention. Several measures for such complex health utility gains have been devised. The notion of QALY, quality adjusted life year, is well-known today by all health economists. It is composed of the two parts, QA, “quality of life” and LY, “life year”. These QALYs form the ‘return’ part of the investment and money forms the ‘cost’ part of the intervention.

The cost-effectiveness ratio is obtained by adequately discounting and subsequently comparing the two: present discounted costs and present discounted effectiveness.

An alternative attempt to measure health is to recognise that health cannot realistically improve indefinitely. Some economists thus made attempts in minimising *disutility*, i.e. by how much our health state fall short of perfect health. A recent concept towards this has been the introduction of the “burden of disease”. These are measured, often, using DALYs, disability adjusted life years, introduced in the early 1990’s. The DALY differs from the QALY primarily in that it includes an age-weighting factor and represents a negative quantity (to be minimised rather than QALYs that are maximised) (Sassi, 2006). These DALYs would be discounted at their respective rates and the best program would be chosen. However, the issue always remains that by discounting health effects through time, the weights of future health gains from the vaccination program are effectively decreased compared to those of the curative intervention. As a result, it has been suggested that vaccines might warrant a different approach.

Recently, a novel measure for the return part of the cost-effectiveness analysis has been proposed. Bos *et al* (2005) argued that risk reduction can be translated into an effectiveness measure in the cost-effectiveness ratio as risk reduction will ultimately reflect a rise in life expectancy. Such an approach takes into account the fact that the instant that health gains occur needs not coincide with the instant that risk reduction occurs and also takes into account other benefits such as herd immunity. They proposed to discount the health benefits of vaccines only for the period between vaccination time and the moment that the infection is prevented. The time between infection and actual health gains is not discounted. This ‘time-shifted’ discounting approach is discussed, in more detail, in section 2.2. Such attempts are purposefully to improve outcomes of evaluations of preventive interventions.

2.1.2 Ethical CEAs

While the dominant equal discounting of costs and effects is firmly based in normative economic theory, the fact that ICERs of preventive interventions are especially influenced by discounting, has led some to argue for specific discounting rules for preventive interventions; aiming to limit the *devaluation* of future health gains (Tasset *et al*, 1999). Beutels *et al* (2008), for example suggested that vaccines might warrant the application of a different discounting approach for health outcomes. Although a lower discount rate for health is generally argued to lack the theoretical motivations, attempts have been consideration of a rate that, at least, decreases with time. Several approaches have been proposed in this context. Step-wise discounting (which may be viewed as a discontinuous form of hyperbolic discounting), for example, starts with discount rates that discontinuously decrease after specific time intervals. Other authors have argued that empirically observed rates of time preference would be more informative (Asenso-Boadi *et al*, 2008, Bobinac *et al*, 2011, Cairns , 1994, Lazaro , 2002, Olsen , 1993, West *et al*, 2003).

Importantly, it is repeatedly observed that discount rates in empirical studies decrease with increasing time horizon, potentially approaching, for example, a hyperbolic function (Harvey , 1995, Loewenstein and Prelec , 1992). This is generally the case regardless if respondents are explicitly asked to adopt a societal perspective or an individual perspective (Asenso-Boadi *et al*, 2008, Bobinac *et al*, 2011, Meerding *et al*, 2010). Hyperbolic discounting generally reflects preferences of individuals that are impatient today but project to become more patient in the future, which is the common observed behavior. It is, however, important to note that observed discount rates are typically high; consequently using such alternatives would result in less favourable cost-effectiveness ratios of preventive interventions, especially since the discount rates for costs are usually given and fixed. Discounting of costs is fairly undisputed in the context of economic evaluations. Often, the exact discount rate for

costs is informed by real returns on riskless government bonds. Table 2.1, below, provides discount rates for costs and health from various guidelines.

Table 2.1: Country specific discount rates for costs and health

Country	Discount rate	
	Costs	Health
Austria	5%	5%
Belgium	3%	1.5%
Canada	5%	5%
England & Wales	3.5%	3.5%
France	0, 3, 5%	0, 3, 5%
Germany	5%	5%
Switzerland	2.5, 5, 10%	2.5, 5, 10%
Sweden	3%	3%
The Netherlands	4%	1.5%
United States	3%	3%

With regards to empirical discount rates, health economists are more concerned with the general decline of the rate function with time rather than the level of discounting. Declining rates suggest that ethical norms could be settled, although not with regards to the near future but to the distant one. Therefore, it has been recommended to use such discount functions for informative purposes in economic evaluations (for example, in sensitivity analyses), also in the Dutch context (Rutten-van Molken *et al*, 2000). Such choices can obviously impact relevantly on cost-effectiveness ratios of preventive interventions (Cairns and van der Pol, 1997). Preventive interventions, especially those with outcomes occurring furthest in time compared to timing of costs, are the most affected by discounting of health gains. Human Papillomavirus (HPV) vaccination can be considered as one of the most extreme examples of such a program, where the major health gains occur approximately 30 years after initial vaccination.

2.2 An extreme example

Since averted cases of cervical cancer occur about three decades after the vaccination, cost-effectiveness of HPV-vaccination has recently gained quite some attention in many countries regarding whether or not to fund (large scale) vaccinations. Attempts to improve outcomes of health evaluation have, however, been numerous. Although this intervention is disease specific, different discounting approaches can nonetheless importantly indicate the weight that future health outcomes of vaccination programmes receive in an evaluation (Mangen *et al*, 2010, Rogoza *et al*, 2009).

This variation should provide the reader with a clear illustration of the impact of adopting different approaches to discounting health effects. Because the health outcomes related to HPV vaccination are expected to occur several decades after the initial vaccination, the number of QALYs gained by HPV vaccination is highly sensitive to the discounting method that is applied (Cairns , 2006, Rogoza *et al*, 2009). In the current example, my colleagues and I illustrate the impact of different guidelines on future health outcomes and, consequently, on the cost-effectiveness of HPV vaccination.

2.2.1 Overview of approaches

For the purpose of our illustration, we shall use a previously published in-house Markov model for HPV-infection (Rogoza *et al*, 2009). We investigate five different discounting approaches, and focus on changing discount rates, and approaches for health effects. To keep the results tractable, we will not vary the discount rate for costs in the current study (which will be set at 4% according to Dutch guidelines), but primarily highlight differences in the NPV of health effects based on different discount approaches and rates. In setting specific limits to the range of discount rates, we stayed in line with the Dutch guidelines. Below we highlight the different approaches used for QALYs in this study.

Constant discounting approach

Constant discounting approach is well-founded in economic theory, and reflects a generally accepted and commonly used discount rule for future costs and health outcomes in health-economic evaluations (Keeler and Cretin, 1983). Letting $w(t)$ be the weight attached to an outcome occurring at time t and r be the *constant* discount rate per unit time², the discount weight is given by

$$w(t) = \frac{1}{(1 + r)^t} \quad (2.1)$$

By using the constant discounting approach, future costs and health outcomes are devalued at a constant rate from the moment that the intervention (e.g. the vaccination) took place. A discount rate for costs of 3-5% is most often used, internationally. Most countries prescribe an equal discount rate for health effects (e.g. 4% for costs and 4% for effects). The reason for this is especially to avoid inconsistencies (Claxton *et al*, 2006, Keeler and Cretin, 1983). Moreover, health effects represent a monetary value, and therefore, applying two different discount rates has often been dismissed as being inconsistent. I will, however, discuss the consistency argument, in more detail, in chapter 6.

Empirical discounting approaches

In contrast to the constant discounting approach, empirical studies typically show that the rate of time preference declines over time, both from an individual and a societal perspective (Cairns and van der Pol , 1997, van der Pol and Cairns , 2000, 2001). This was recently confirmed for health effects, as well, in a meta-regression analysis by Asenso-Boadi *et al* (2008). In particular, the rate of time preference for a short-term delay (e.g. 5-year) was approximately 25% which decreased to approximately 3.5% for a long-term delay (e.g. 100-year). Such alternative discounting approaches have been

²Yearly time intervals are most common.

proposed to better reflect observed time preference. Two prominent examples of such decreasing rates are

hyperbolic discounting

$$w(t) = \left(\frac{1}{(1 + gt)^{h/g}} \right) \quad (2.2)$$

and proportional discounting

$$w(t) = \left(\frac{b}{b + t} \right)^\gamma \quad (2.3)$$

where h, g, b and γ are parameters reflecting the time preference for the future (Cairns and van der Pol , 1997, Harvey , 1995, Loewenstein and Prelec , 1992). In equation 2.2, h , reflects the individual's preference for the future or timing in general. An individual does not have any time preference (i.e. a discount rate of 0) if $h = 0$; by increasing h the preference for the present increases. Parameter g determines how much the function differs from the constant discounting model (van der Pol and Cairns , 2010), with $g = 1$ resulting in the constant discount model. Proportional discounting (equation 2.3) has been proposed by Harvey (1995). The parameter, b , reflects the magnitude of the time preference and γ determines the shape of the curve. Initially, Harvey suggested that γ should be 1 but others have introduced different values for γ . For example, Cairns and van der Pol (1997) estimated that the proportional discounting model would fit empirical data best if γ is 1.5.

We will fit the proportional and hyperbolic discounting approaches by varying the values of the variables to minimize the sum of the squares and maximising the explanatory power (reflected in r^2) as reported by Asenso-Boadi *et al* (2008). In particular, for the hyperbolic discounting approach the values of h and g are estimated at 0.32 and 0.29 ($r^2 = 0.9988$), respectively. For the proportional discounting approach the values for b and γ are estimated at 3.4 and 1.1 ($r^2 = 0.9988$), respectively.

Stepwise approaches

In the step-wise discounting approach, several constant discount rates are used in decreasing order for different periods. Typically, step-wise begins with a constant rate but this is lowered in consecutive time periods (The graph of rate against time looks like a flight of stairs). Table 2.2 shows the discount rates and time intervals as mentioned by Bazelon (2002), Beutels *et al* (2008) and the UK Treasury. With regards to the latter, note that in the static HPV model that we used, the time horizon of our analyses was set at 100 years, and so the minimum discount rate applied will be 2.5%.

Table 2.2: Step-wise discounting approaches proposed in literature

UK Treasury		Beutels <i>et al</i> (2008)		Bazelon (2002)	
Time(Years)	$r(\%)$	Time(Years)	$r(\%)$	Time(years)	$r(\%)$
0 to 30	3.5%	1 to 5	4%	0 to 10	3.5%
31 to 75	3.0%	6 to 25	3%	11 to 20	1.5%
76 to 125	2.5%	26 to 75	2%	> 20	0%
126 to 200	2.0%	76 to 300	1%		
201 to 300	1.5%	> 300	0%		
> 300	1.0%				

Discount rates and time intervals highly differ among different sources. For the UK, the time intervals after which the discount rate decreases with 0.5% were based on empirical data (Oxera , 2002), and a normative framework was the starting point of the analysis. The other two papers were more focused on proposing methods to ‘improve the outcomes’ of economic evaluations of preventive programmes. Bazelon (2002) recently proposed to decrease the discount rate relatively rapidly, so that more weight would be attached to future health. In their proposal, the discount rate reaches 1.5% after no more than 11 years. As a comparison, in the UK Treasury (2013), this discount rate was not reached until after 200 years.

Time shifted approach

Specifically for vaccines, also in an attempt to improve outcomes of evaluations of preventive interventions, it has been proposed that the health outcomes might be discounted from the moment of risk reduction (i.e. averted infections) instead of from the moment that health is actually gained (Bos *et al*, 2004, Drummond *et al*, 2007). Bos *et al* (2004) argued that, in the case of a vaccination programme, the health outcomes of preventing an infection is undervalued due to discounting because, for some infectious diseases, the delay between the initial infection and disease development can be several years. Therefore, they recommended the time shifted discounting approach, by which health outcomes are discounted from the moment the infection was prevented and not from the moment, for example, life years are actually gained.

Although this method has been used by others as a pragmatic discounting approach, an exact underpinned normative rationale for it has not been given. Furthermore, one might argue that by using this discounting approach, some basic principles of discounting, such as accounting for the uncertainty in the period after the infection is prevented, are violated. We used this method with two different discount rates in the period where discounting is required according to the method; a 4% discount rate and a 1.5% discount rate. QALY losses due to cervical cancer were thus only discounted in the period between vaccination time and the moment of HPV-infection.

2.2.2 Results

All five discounting approaches were applied to the health outcomes of our in-house Dutch HPV model. This model predicts the incidence of cervical intraepithelial neoplasia and cervical cancer incidence with and without HPV vaccination, reflecting the current Dutch situation. The implementation of HPV vaccination for the full cohort of 12-year-old Dutch girls (i.e., cohort size was set at

Table 2.3: Overview of different discounting approaches introduced above

Approach	Name	Description
1	Constant	Discount rate is constant over time (equation 2.1), can be uniform or differential
2	Step-wise	Discount rate step-wise declines after specific time intervals (table 2)
3	Hyperbolic	Discount rate declines gradually over time (equation 2.2)
4	Proportional	Discount rate declines gradually over time (equation 2.3)
5	Time-shifted	Time period between vaccination and prevention of infection is discounted rather than full period up to actual QALY gains

100,000) resulted in an undiscounted lifetime gain of 2907 life-years or 3462 QALYs. The total undiscounted costs of implementing HPV vaccination to the Dutch National Immunization program (Rijksvaccinatieprogramma) were €31.5 million (€30.9 million discounted) and resulted in €11.5 million undiscounted cost offsets (€2.8 million discounted).

Applying the different discounting approaches, of course, resulted in different numbers of discounted QALYs gained by HPV vaccination (Table 2.4). The time-shifted discounting approach resulted in the highest present value of QALYs while the proportional discounting approach resulted in lowest estimate. Obviously, this result is driven by the relatively high discount rates in the latter method (i.e. much higher than 4%). In the time shifted discounting approach (Bos *et al*, 2004) , health outcomes of HPV vaccination were only discounted at a constant rate for the period between vaccination and infection, and in other periods a zero discount rate was applied.

When the stepwise discounting approach was applied, the total num-

ber of discounted QALYs were comparable to those obtained at a constant rate of 3%. Note that the total number of discounted QALYs is sensitive to the time interval and decline in discount rate. When the conventional constant discounting approach was applied, the present value of QALYs gained was highly sensitive to the chosen discount rate. Lower discount rates for health outcomes resulted in substantial increases in total number of discounted QALYs gained with HPV vaccination.

To give an indication of the impact of these different approaches on the final ICER, we also combined these results with the discounted costs (4%). It should be noted that we do this mainly for illustrative purposes and that it might not always be logical to combine our results on health gains with a cost-estimate, discounted using a 4% constant rate. In particular, if one prefers an empirically based approach such as hyperbolic discounting for health effects, it is likely that one also wishes to discount costs on a similar basis, that is, using a hyperbolic discount function. The results on QALYs and ICERs are, nonetheless, shown below.

Table 2.4: Discounted health outcomes of HPV vaccination

Discounting approach	QALYs gained	ICER(€/QALY)
Undiscounted	3462	7600
Constant 1.5%	1423	18400
Constant 3%	715	37000
Constant 4%	438	59100
Proportional	164	165400
Hyperbolic	160	164500
Step-wise*	718	36800
Shifted uniform 4%	2117	13200
Shifted differential 1.5%	2811	9400

HPV= human papillomavirus; ICER= incremental cost-effectiveness ratio; QALY=quality adjusted life-year

* As proposed by the UK Treasury.

According to Dutch guidelines (constant discount rates of 4% for money and 1.5% for health; i.e. differential discounting between money and health), we found an ICER of €18,400 per QALY gained for HPV vaccination. Furthermore, varying the discount rate for health effects from 0% to 4% resulted in estimated ICERs of €7,600 to €59,100 per QALY, still using constant discounting. When we applied the proportional discounting approach to health effects, we found an ICER that was nine times higher than the benchmark of 18,400 per QALY gained. Ergo, extremely large and relevant differences in the ICER were found between the various approaches investigated, moving from extremely cost-effective up to extremely cost-ineffective (when compared with commonly cited thresholds).

2.2.3 Discussion

Proportional and hyperbolic discounting approaches are important as they reflect individual and societal time preference for health outcomes (Cairns and van der Pol, 1997). Although their use cannot be acknowledged without context, it is informative to apply them more frequently in economic evaluations, for instance in sensitivity analyses (Bleichrodt and Brouwer, 2000). This is also interesting, since the approach of differential discounting, as prescribed in the Netherlands, and the proportional and hyperbolic discounting functions, more or less, represent extremes, in terms of outcomes. Thus, their use in sensitivity analysis would be informative for decision makers.

The different methods, however, highlight the issues and ongoing discussions in the literature. For these alternative discounting approaches, the devaluation of the health outcomes was both dependent on the parameter values used in the discounting approaches and the nature of the approaches themselves. Obviously, when the health outcomes were devalued less, this resulted in a more favourable cost-effectiveness for HPV vaccination. The recommendation to carefully identify appropriate discount methods and rates, and to investigate the sensitivity of results to the application of alternative dis-

counting approaches, holds even stronger if vaccination programs are considered with health effects occurring far into the future.

2.3 Motivation matters

Although discounting health is subject to ethical debates (Bos *et al*, 2005, Brouwer *et al*, 2005, Brouwer and van Exel , 2004, Claxton *et al*, 2006, Coyle and Tolley , 1992, Frederick *et al*, 2002, Gravelle *et al* , 2007, Keeler and Cretin, 1983, Klok *et al*, 2005, van Ballegooijen *et al*, 2010, Weinstein and Stason, 1977), most countries suggest to discount effects with a constant discount rate equal to that of costs (commonly of around 3-5%) (ISPOR Guidelines , 2012, Smith and Gravelle , 2001). In general, it seems important for decision making and guideline prescribing bodies in different jurisdictions to have transparent, defensible and reasoned discount rules. The constant discounting approach is well founded in economic theory, and remains the generally accepted and recommended discounting approach. It (purposely) does not reflect commonly observed declining time preference of individuals, as this avoids time inconsistent behaviour in policy making.

While the different discounting approaches are informative, it must be noted, however, that some approaches have a better normative underpinning (e.g. constant uniform) while others are based, solely, on empirical studies (hyperbolic and proportional discounting). Some other approaches (e.g. the stepwise approaches proposed by Beutels and colleagues and that proposed by Bazelon and colleagues, as well as the shifted approach by Bos and colleagues) lack both yet (Bazelon , 2002, Beutels *et al*, 2008, Bos *et al*, 2004). Their motivations seem especially based on the dissatisfaction with the ‘devaluation’ of future health in economic evaluations of preventive interventions. While this may be a poor motivation for alternative methods, it does highlight potential problems in the standard method that assumes a constant discount rate, equal for both costs and health

outcomes. In general, it seems inappropriate to change the methodology applied in evaluations on such a basis. Therefore, as consensus would have it, majority of bodies prescribe the standard model that assumes an invariant discount rate.

2.a Appendix: Pathogenesis cervical cancer (Stanley (2010))

Cervical cancer is the second most common cancer worldwide. Infection with the Human PapillomaVirus (HPV) is a prerequisite for cervical cancer, and the persistence of the infection is especially important. In particular, infection with one of the oncogenic types of HPV may develop into cervical intraepithelial neoplasia (CIN) of grades I III and ultimately into invasive cancer. Major oncogenic serotypes are 16, 18, 31, 33, 45, and 52. Of these serotypes, HPV 16 and 18 have shown to be responsible for approximately 70% of cervical cancer cases worldwide.

In the Netherlands, HPV infection peaks are found in women aged 20 25 years. Although most women are able to clear the infection within one year, some of them will develop persistent infection. Women can develop CIN I III and cervical cancer after some years of persistent infection. In the Netherlands the average age of cervical cancer is estimated between 40 45 years.

Currently, highly effective prophylactic HPV vaccines are available. HPV vaccines are most effective if administered to women who are HPV negative. Therefore, women should be vaccinated before they become sexually active. Most developed countries decided to implement HPV vaccination of girls aged 12-years in National immunization programmes.

Health benefits following HPV-vaccination comprise of prevention of cervical intraepithelial neoplasia or development of cervical cancer. These health benefits are expected to occur approximately 20 and 30 years after the initial immunization, respectively.

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Chapter 3

The Social Discount Rate

¹ The use of a unique constant discount rate for all types of consumption is a central assumption in the standard model of intertemporal choice, the discounted-utility (*DU*) model, first introduced by Samuelson (1937). While children are familiar with the concept of interest rate, a term that economists often use is the socially efficient discount rate. Socially efficient discount rates are proxied in three different manners: the rate observed in financial markets revealing society's willingness to transfer wealth to the future, the marginal rate of return on productive capital in the economy and the welfare-preserving rate of return on savings. The three different proxies are, however, known to be fully compatible with each other when consumption plans are optimized and credit markets are frictionless (Gollier, 2012). Such simplifying assumptions are ubiquitous internationally. In order to ensure proper motivations of their projects, international guidelines for economic evaluations prescribe a devaluation of future benefits, monetary or non-monetary, using such a rate.

¹Based on

PAROUTY MBY, BOERSMA C AND POSTMA MJ (2013) *Discounting human lives: A critique* submitted

3.1 The Ramsey discount rate

Majority of economists have adopted the Ramsey (1928) discount rate, r , for that matter. Ramsey (1928) assumed that any individual should aim to maximise his/her utility. He considered the two possible scenarios; indefinitely increasing enjoyments or enjoyment rate that asymptotically approaches a certain fixed limit. It seemed rather logical that utility cannot increase indefinitely and the preferred assumption was that an individual should aim to attain *Bliss*. With a given income, which is a function of labour and capital, an individual can only consume now or save for future consumption and the whole problem was to seek the optimum balance between the two.

3.1.1 Ramsey brief

Briefly, the intuition behind Ramsey's calibration of r is as follows. Suppose that we were given a bag full of grapes and were to devise an optimal consumption plan for our primate friend, *Saguinus oedipus*. The grapes can be consumed immediately, or they can be planted to produce trees, yielding more grapes, in the future; i.e. grapes are a form of capital yielding a benefit for the future. Moreover, if s grapes are saved for the future, then they yield $f(s, T)$ grapes in the future. Suppose that an initial cost s units of grapes per capita will yield a sure benefit $R = s(1 + r_d)$ units of grapes per capita after 1 time period, where r_d is the rate per unit time. A natural continuous function for $f(\cdot)$ is thus obtained:

$$\begin{aligned} f(s, T) &= s(1 + r_d)^T \\ &= s \lim_{\Delta T \rightarrow 0} (1 + r_d \Delta T)^{\frac{T}{\Delta T}} \\ &= s e^{r_d T} \end{aligned}$$

where r is the continuously compounded growth rate. Since a unit of grape per capita invested in the primate feeding project, will yield e^{rT} units of grapes per capita in the future, with certainty, we can

transfer grapes through time by investing in our own primate feeding project. Thus, a simple arbitrage argument deduces:

$$f'(s) = e^{rT} \quad (3.1)$$

Now, let q^0 and q^T denote the quantity of consumption today and in the future, respectively. The primate can be assumed to derive pleasure solely by consuming and it evaluates its lifetime utility as $U(q^0, q^T)$. Assuming that we are given an initial endowment of $q^0 = Q^0$ grapes today, if s grapes are planted, the primate can consume $Q^0 - s$ immediately. The quest to find how much of our grapes we should plant is similar to the problem that Ramsey wished to tackle: *"How much of its income should a Nation save"*?

Supposing that the primate consumes $Q^0 - s$, then the lifetime utility is given by $U(Q^0 - s, f(s, T))$. It is generally assumed that, given one riskless asset and one risky asset, the fraction of wealth optimally placed in the risky asset is independent of the level of initial wealth. Such an assumption hardly seems unacceptable. The utility function, $U(\cdot)$, thus assumes that the relative risk aversion is constant. The only utility function with such a feature is the isoelastic utility function:

$$U(q) = \frac{q^{1-\varepsilon}}{1-\varepsilon} \quad (3.2)$$

Now, the optimal consumption plan is characterized by the tangency of the feasibility frontier (the set of feasible consumption in both periods) and the indifference curve. Then, the increase in future consumption when one extra unit of grape is invested in the productive capital of the primate feeding project, measured by the gradient of the feasibility frontier, should equal the marginal rate of substitution between current and future consumption at the optimum consumption plan. Also, because of decreasing marginal productivity of capital, the feasibility frontier is concave. The first order condition of the problem of maximizing $U(Q^0 - s, f(s))$ with respect to s is thus $f'(s) = \frac{\partial U(q^0, q^T) / \partial q^0}{\partial U(q^0, q^T) / \partial q^T}$.

Ramsey, however, did not advocate a rate for impatience; a famous quote from Ramsey (1928) reads as follows: *"It is assumed that we do not discount later enjoyments in comparison with earlier ones, a practice which is ethically indefensible and arises merely from the weakness of the imagination"*. Years following Ramsey's paper, however, economists included such a rate. A technical definition for the latter is the minimum interest rate that induces people to save when their income profile is flat; i.e. the rate of impatience, ρ , serves merely to value a future consumption impatience-free. So, our first order condition becomes

$$f'(s) = \frac{\partial U(q^0, q^T)/\partial q^0}{e^{-\rho T} \partial U(q^0, q^T)/\partial q^T} \quad (3.3)$$

We now bring the different equations of this section together. We first equate 3.3 with 3.1 and then plug in the parameters from 3.2 and we are done. That is

$$\begin{aligned} e^{rT} &= \frac{\partial U(q^0, q^T)/\partial q^0}{e^{-\rho T} \partial U(q^0, q^T)/\partial q^T} \\ \longrightarrow r &= \rho - \frac{1}{T} \ln \frac{\partial U(q^0, q^T)/\partial q^0}{\partial U(q^0, q^T)/\partial q^T} \end{aligned} \quad (3.4)$$

From equation 3.2, we find $U'(q) = q^{-\varepsilon}$. So,

$$r = \rho - \frac{1}{T} \ln \left(\frac{Q^T}{Q^0} \right)^{-\varepsilon}$$

Defining the growth rate of consumption between dates 0 and T to be g so that $Q^T = Q^0 e^{gT}$, we can write

$$r = \rho + g\varepsilon \quad (3.5)$$

NICE (2008), for example, prescribes $r = 3.5\%$ for both costs and health effects, based on the allocation of the numericals $\rho = 1.5\%$, $g = 2\%$ and $\varepsilon = 1$, recommended by the UK Treasury (2013).

3.2 Ramsey rate for health?

Given that savings receive interest r on capital, Ramsey showed that optimum saving is given by the simple rule: “*The rate of saving multiplied by the marginal utility of money should always equal to the amount by which the total net rate of enjoyment of utility falls short of the maximum possible rate of enjoyment*”. Therefore, assuming economic growth, an individual should aim in finding the best proportion between current consumption and future saving. With Ramsey’s assumptions being within realistic realm, it seems that there is need for discounting to take into account the marginal substitution between the present and the future.

However, when health effects are concerned, our valuations tend to require value judgements from both moral and philosophical points of views. Past philosophers have often argued that “*life is priceless*”. Yet, valuing a human life in monetary terms is, in some way, tagging a price onto our lives. While a zero discount rate for health implies that similar weights be attached to both current and future health effects, a procedure which seemingly has the acceptable required ethics, decision makers would argue that programs with large annual costs and moderate health effects would also appear cost-effective since the weights assigned would favour ‘effectiveness’ more than ‘costs’.

This could inflate the portfolio of feasible interventions thus raising the opportunity costs of non-selected programs. Noting that decision maker’s major role to be an efficient resource allocator entails distribution of resources such that output is maximised per unit cost while minimising opportunity costs of non-selected programs, it would seem that priority setting difficulties might then be expected. As a means to aid priority setting, given a fixed budget constraint, the discount rate of health outcomes that is typically prescribed generally equals to that of costs (such as, for example, NICE (2008)) given the assumption of invariance in the DU -model and the marginal substi-

tution between health and wealth.

3.2.1 The marginal substitution between health and wealth

As with the discounted utility model (Frederick *et al*, 2002), the theoretical framework for discounting health is however largely “*due to its simplicity and it’s resemblance to the familiar compound interest formula, and not as a result of empirical research demonstrating its validity*”. One of the fundamental financial principles states: “A dollar today is worth more than a dollar tomorrow”. Whether the same principle holds true for health effects is, however, debatable.

When we consider a financial bond, for example, yielding a certain amount of dividends in the future, the quantitative measurability of the income streams generally do not pose any problems in the discounting procedures. However, there seem to be quite a lack of consensus regarding the very quantification of health effects. The general topic of discounting health effects consists mainly of two parts namely a methodological part which is derived mostly from the financial literature and a subjective part which is much debated. Economists such as Brouwer and van Exel (2004), for example, argue that we first of all have to create consensus on qualitative decision rules rather than on the actual rates.

Several investigations have been carried out as an attempt to understanding the rates of preference for health and wealth and their exact relationship (Cairns and Van Der Pol, 1997, Chapman and Elstein, 1995, Viscusi, 1996). The marginal transferability of wealth and health seems to be a touchstone for the choice of the discount rate to be applied for health. Money is generally known by economists as the most liquid asset. But assuming a perfect transferability would also imply that health is as liquid as money. A prime requirement in investigating the relationship between health and wealth is thus the very liquidity of health effects.

The marginal substitution between health measures and money thus appears questionable. Recently, Robert Stonebraker (<http://faculty.winthrop.edu/stonebrakerr/book.htm>) stated that "*If the price of a hamburger rises, I switch to chicken substitution is the key*". When goods are mispriced, the law of demand and supply forces the price to stabilise so that all goods are in their proper respective equilibrium. But whether health effects have got any substitutes or whether health can actually be priced is a matter of delicate moral values and philosophy.

3.3 A differential rate for health

The specific nature of a human life, as opposed to other commodities includes the indivisibility of a life year in a sequence of life years gained and the dependence of life years within the sequence. In particular, one cannot live an individual life year without living the previous one; i.e. the subsequent life year is dependent on the previously lived ones and, as such, life lived is indivisible (It is not without reason that one refers to "*a life*" because, for example, one cannot live half of the previous life year) (Bos *et al*, 2006). If life years are dependent and indivisible it would not be adequate to discount a sequence of life years gained by one individual year by year, rather, one ought to discount the whole composite of accumulated years once, from the start of the sequence back to the moment of implementation of the project being investigated. Such an aspect again motivates the use of a decreasing or even zero discount rate within the sequence of dependent life years.

The specificities of health thus do not allow for an assumption of a single rate. Although attempts to aid the ethical valuation of future health have been propositions of different discounting approaches, seen in chapter 2, the lack of motivation for differential discounting has, thus far, mirrored on a lack of consensus. Most governments

suggest a one and only discount rate for both health and costs. Some authors argue that, in order to be consistent, health and wealth being proxies for each other, require similar discount rates for both (Goodin, 1982, Keeler and Cretin, 1983, Parsonage and Neuburger, 1992).

3.3.1 Components of the Ramsey rate

Assuming the proxy argument, let us investigate the validity of the famous Ramsey (1928) rate for health. Since higher income is generally a proxy for better health (an assumption which is however debatable), economists have suggested transferring the Ramsey discount rate to health which implies transferring the components of the Ramsey rate to health measures, as well. His Graph from "*A mathematical theory of saving*" is reproduced below.

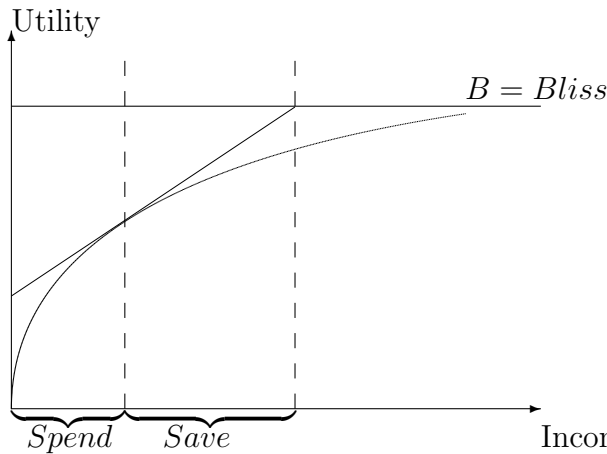


Figure illustrating what proportion of income one should save so as to indefinitely be attempting to approach bliss. Note that the graph is an increasing one. (i.e. as Ramsey assumed: "no misfortunes will occur to sweep away accumulations at any point in the relevant future.")

Figure 3.1: Graph from Ramsey's "*Mathematical theory of saving*"

We recall equation 3.5, $r = \rho + g\varepsilon$, where the three components are

ρ , the rate of pure time preference, g , the growth rate of per-capita consumption and ε , the elasticity of marginal utility of consumption. The pure rate of time preference simply reflects the degree of impatience; everything else being equal (i.e. $g = 0$). It is the marginal rate of substitution between present and future so that consumption levels in both periods are *equal*. And the factor $g\varepsilon$ reflects the growth rate of happiness from consumption, measured in terms of utility.

The 'pure' rate

The extent to which we should assign weight to future and present generations in our moral decisions has been much explored since Ramsey. Kavka (1978) provided an elegant argument towards the rights of future generations. He condensed the major debates into three main reasons for a positive pure rate of discount: the intertemporal location of the people, our knowledge of them and the contingency of their lives.

The first reason has been traditionally dismissible by most authors including Ramsey. Sidgwick (1874), for example, pointed out that the maximization problem in utilitarianism yielding "*the greatest happiness of the greatest number*" should not exclude distant generations. The second reason pertains mostly to the needs and desires of future generations. The trivial argument that we don't know the demands of future generations has been intellectually much contended. Kavka, for example, argued that enough food to eat, air to breathe, space to move and fuel to run our machines are definitely known prerequisites. And we are left with exploring the third reason.

Dasgupta and Heal (1979) argued that a non-zero probability of extinction of the human species might justify a positive pure discount rate. A concise description of their derivation (Pontherie, 2003) goes as follows: Suppose a rational individual, belonging to an unknown generation, faces the choice of an optimal infinite consump-

tion stream for society. According to Laplace's Principle of Insufficient Reason, the individual should assign equal probability to the existence of each generation. However, considering an infinite time horizon, "equi-probability" would make no sense and Dasgupta and Heal introduced an immediate subjective probability that human life on the Earth will cease beyond a time t . Assuming that this probability is decreasing in t , then priority given to existing people with scarce resources is justified. Although this "probabilistic discount rate" is different from the "temporal discount rate" (Parfit, 1984), the decrease in t , which posits that the further generations are less likely to exist than the closer ones, indicate some correlation to the temporal rate which reflects the importance of future generations (Ponthiere, 2003).

In any case, nonexistence of future generation warrants a non-zero positive pure discount rate for a society with scarce resources; i.e. the pure rate is defensible depending on existence of future lives and that rate is applied to utility from scarce resources. Hence, since the theoretical foundation towards incorporation of the pure discount rate is the non-existence of future lives but the concept of discounting future lives implicitly assumes their existence, it would seem that inclusion of the pure rate in the discount rate for health is circular reasoning. Consequently, discount rates prescribed for health in most policies are over-estimated.

As a result, incorporation of the pure rate in discounting future lives does not seem to have the theoretical foundations. Since discounting future lives assumes their existence, in cases of the use of the social discount rate for both life years and costs, it appears that rationality dictates that one should omit the pure rate in the discounting of life years. Moreover, as the QALY is a multiplicative combination of life years and quality of life, such zero pure-rate might as well be applicable to other health measures. In order to avoid circular reasoning, I am inclined towards differential discounting for health and costs, omitting the pure rate for health.

The growth rate of consumption

Moreover, economists are much more heroic in extending the assumption of economic growth to QALYs or life years or other health measures. As Ramsey himself stated, “*The most serious factor neglected is the possibility of future wars and earthquakes destroying our accumulations*”. Although life expectancy, or life duration, seems to be correlated with economic growth, the issue is that an individual expects income to grow as she ages but unfortunately expects health, or health quality, to decline. Such assumptions challenge the foundations for an equal rate for health and wealth.

Nonetheless, assuming decisions for society, as a whole, one might argue that growth in life expectancy should be enough of our consideration. Even so, growth of life expectancy is generally moderate compared to economic growth. The effects (estimated in log terms) of a 1% increase in life expectancy on economic growth, recently provided by Husain (2012), are: 4.2% in Barro (1996); 7.3% in Barro and Lee (1996); 5.8% in Barro and Sala-I-Martin (1995); 6.3% in Bloom, Canning and Malaney (2000); 3.7% in Bloom and Sachs (1998); 4% in Bloom and Williamson (1998); 3% in Gallup and Sachs (2000); 7.2% in Hamoudi and Sachs (1999) and so on. As a result, it would seem that the parameter, g , the growth rate of life expectancy would be lower than that of economic growth.

Elasticity of marginal utility of consumption

Noting that Ramsey’s derivation considered only 2 time periods: “now” and “the future” where the future is seen as a single period under consideration, the parameter, ε , is also challenged. Valuation of health effects is done in a bipartite way: namely the life years or life duration and the value of a single life year. The utility model for health is typically characterised by, firstly, linearity; i.e. the utility attached to a life year gained is independent of the timing of occurrence and, secondly, by additivity; i.e. the utility of survival is ad-

ditive over time (Miyamoto, 1999). Thus, the utility model for life years should, not only be an isoelastic utility function, but should have the same level of elasticity in order to admit the invariant Ramsey rate.

3.4 Discussion

Majority of bodies in today's modern societies recommend a discount rate for health that equals that of costs. However, a discount rate for human lives equalling that of money does not seem to build on a well reasoned theory. First, assumptions of marginal substitution between health and wealth do not seem valid due to the very nature of a human life. Second, it appears that an ethically defensible discount rate for health necessarily ought to be lower than that of costs. Impatience is a psychological trait that should be in our economic theory; however it does not appear ethically correct to include such a rate in the discount rate of health.

Experts in the field are even puzzled by the appropriateness of impatience for the very evaluation of social welfare. Arrow (1999) and Gollier (2012) provide a nice compilation of such citations. Sidgwick (1874): *"It seems ... clear that the time at which a man exists cannot affect the value of his happiness from a universal point of view; and that the interests of posterity must concern a Utilitarian as much as those of his contemporaries"*, Koopmans (1960): *"[I have] an ethical preference for neutrality as between the welfare of different generations"*, Solow (1974): *"In solemn conclave assembled, so to speak, we ought to act as if the social rate of pure time preference were zero"*, Harrod (1948): *"Pure time preference [is] a polite expression for rapacity and the conquest of reason by passion"*, to name a few. With regards to discounting human lives, when the definitions of consumer and consume overlap, one ought to revise one's assumptions.

Moreover, given that growth rate in life expectancy is typically lower than that of GDP and the utility function for health needs to be isoelastic in order to yield a Ramsey rate, it would appear that, the commonly assumed discount rate for health, $r = \rho + g\varepsilon$, is not theoretically defensible. As a result, recalling that, in Ramsey's derivation, the individual's objective was to maximise his/her utility, then with regards to human lives, we argue, that the government objective should be redesigned. Nonetheless, inclusion of the pure rate is circular reasoning, growth in life expectancy is generally lower than growth in GDP and the utility model assumed for a human life ought to be isoelastic wherefore a theoretically defensible discount rate for health should be lower than the *SDR*, implying differential discounting.

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Chapter 4

Questionnaire Development

¹ The theoretical indefensibility of an equal rate offer presumptive reasons to suspect that empirically observed rates for health outcomes and for money might be different. It appeared natural to carry out an empirical investigation into a differential rate for health compared to the rate for monetary outcomes, such as income. In order to do that, we sought to gather time preference data from a few hundreds of individuals. However, although a word provides a means to a meaning, variations in question formulation of a unique problem have, historically, each proved to have their own right to exist. Johannesson and Johannsson (1997), for example, found that implied time preferences are contingent on the method of elicitation.

Due to divergent elicitation procedures, empirical investigations about time preferences have rarely converged. Framing of the problem is commonly known to yield differing results (Fischhoff *et al*, 1980, Tversky and Simonson, 1993). For example, the assumption that €100 today is worth more than €100 in a year's time imply that the

¹Based on

PAROUTY MBY, KROOSHOF DGM, WESTRA TA, PECHLIVANOGLU P AND POSTMA MJ (2013) *Reviewing and piloting methods for decreasing discount rates* Expert Reviews of Pharmacoeconomics and Outcomes research, accepted

future is valued less than the present. However, individuals generally prefer increasing sequences than decreasing ones. Hence formulation of questions regarding intertemporal choices differentiate time preference studies from sequence studies.

Furthermore, individuals intertemporal preferences are also known to diverge when temporal distances are formulated in terms of dates or delays. Divergences in responses exist when questions are framed in, for example, *in five years time*, or *when you are five years older*, or *five years from now*. With that regards, we first piloted a sample questionnaire among pharmacy students. The pilot sought to elicit questions with considerations on:

1. Consistency of the questions for all types of outcome.
2. Clarity of the questions and how questions would be conceived.
3. The possibility to elicit time preference separately for monetary and health outcomes.
4. The possibility to elicit indifferences in consumption for different time points in the future.
5. The possibility to elicit optimistic or pessimistic views about the future (with regards to both money and health)

4.1 Problem framing

Due to divergences in responses resulting from questionnaire set-ups, we seek a formulation that would, firstly, allow for comparability between discounting money and discounting quality of life. As far as possible, we shall aim to frame our queries in similar fashion for both outcomes. Thus, differing discount rates for dissimilar outcomes, would contrast more. Erecting questionnaires to elicit intertemporal choices generally fall into three main categories; and responses also typically fall into three groups (Frederick, 1999).

4.1.1 Question formulation

Equity inquiry typically asks respondents about different outcomes on a generation basis; i.e. for example, €100 gained by this generation compared to €100 gained by the next. Although we suggest to consider intergenerational equity, comparability of our study to the broad literature investigations implied an alternative formulation. **Sequence inquiry** usually asks respondents to consider increasing or decreasing sequences of outcomes with time. The latter had, however, not much relevance for our purpose. **Explicit inquiry**, as the name suggests, explicitly mentions the gains and the delays. We chose that formulation, as previous studies did before us.

For example, a question from Bobinac *et al* (2011) goes as follows;

You have to choose between:

1. Programme A, which yields 1000 healthy life years among elderly this year. These elderly now have a life expectancy of 79 (with the programme this will be 80).
2. Programme B, which yields 1000 healthy life years among elderly 5 years from now. These elderly now have a life expectancy of 79 (with the programme this will be 80).

In order to allow further comparability to the recent investigation on life expectancy by Bobinac *et al* (2011), delays were also framed akin; i.e. "*in 5 years time*", rather than, say, "*when you are 5 years older*".

4.1.2 Response types

Responses, in time preference investigations, especially with regards to time preferences of life years, typically are choices between two interventions. A **choice** response is generally a preference for, say program A that saves, say, 1000 human lives today compared to program B saving 1000 human lives in 5 years. A **matching** response

type, on the other hand, would state the number of human lives saved so that program *B* feels as good as program *A*. A **rating** response rates a program's appeal compared to another; i.e. when individuals are asked to rate the relative goodness of an outcome compared to another (Svenson and Karlsson, 1989). We included questions that resulted in both choice and rating type of responses. For exact wording of the final questionnaire, please see this chapter's appendix.

4.2 Rule rationality

Responses were verified based on rule rationality. The decision rule of a rational individual has been, rather formally, expressed in literature as statements of consistency among preferences; i.e. axioms of rationality. As a result, responses from the pharmacy students were deemed *good* or *bad* if they met the criteria of *Rational Choice Theory* applied to intertemporal choices.

If we consider the consumption of a single good at two different times, $t = T$ and $t = T'$ as two different goods, say q^T and $q^{T'}$, then the Von Newman-Morgenstern rationality axioms (Von Neumann and Morgenstern, 1953) prove to be quite useful for our purpose. We shall assume that individuals are rational if and only if the following statements hold true regarding their preferences for consumption of q^t , $t = 1, 5, 10, 20$ and 40 . The first axiom, though trivial, states that a decision must be made by the decision maker. Therefore we only considered individuals who *always* expressed some kind of preference for consumption at two different times, either less preferred, \prec , or more preferred, \succ , or indifferent to, \sim , to be deemed rational. As the Von Newman-Morgenstern's first axiom suggests:

Axiom 1. Completeness $\forall T \neq T', q^T \prec q^{T'} \text{ or } q^T \succ q^{T'} \text{ or } q^T \sim q^{T'}$

Secondly, an individual expressing a certain time preference towards a commodity (quality of life or money) should be consistent among

her preferences. That is, suppose an individual expresses more preferences towards consumption in one years time than in, say, five years time and the same individual expresses more preference for consumption in five years' time compared to ten years' time, then that individual is deemed rational if and only if she prefers consumption now compared to consumption in ten years time. Thus, as the second axiom states:

Axiom 2. *Transitivity* $\forall T \neq T' \neq T''$, If $q^T \preceq q^{T'}$ and $q^{T'} \preceq q^{T''}$, then $q^T \preceq q^{T''}$

Thirdly, consider an individual who is consistent among her intertemporal choices and who, for example, prefers consumption in one years time compared to consumption in five years' time and also prefers consumption in five years' time compared to consumption in ten years' time. Then, assuming preferences are continuous functions, that individual should be able to settle for a gamble of consumptions between most preferred(1 years time) and least preferred(10 years time) compared to a sure and certain consumption in 5 years time. There exists an expected consumption with possible outcomes: consumption in 1 years time and consumption in 10 years time, such that the individual is indifferent between a sure and certain consumption in 5 years time to the expected consumption. We therefore have:

Axiom 3. *Continuity* $\forall T \neq T' \neq T''$, If $q^T \preceq q^{T'} \preceq q^{T''}$, then \exists a probability $p \in [0, 1]$ such that $q^{T'} \sim pq^T + (1 - p)q^{T''}$

Lastly, consider an individual who prefers consumption in 1 years time compared to consumption in 10 years time. Then any other consumption bundle that is equally added to the initial set of consumptions should not alter the preferences. That is, if that individual initially prefers consumption in 1 years time to 10 years time and is given the choice between the following two scenarios, A and B, where A=consumption in 1 years time + consumption in 40 years time and B=consumption in 10 years time + consumption in 40 years time, then the rational individual should still prefer A to B because

she initially preferred consumption in 1 year compared to 10 years. The last axiom gives:

Axiom 4. Independence $\forall T \neq T'$, If $q^T \prec q^{T'}$, then for any $q^{T''}$ and $p \in [0, 1]$, $pq^T + (1 - p)q^{T''} \prec pq^{T'} + (1 - p)q^{T''}$

4.3 Preliminary data analysis

Fifty students with an average age of 19 years, participated in the survey. Out of these 50 participants, 2(4%) of the responses from the monetary outcomes and 14(28%) from quality of life analysis violated at least one of the axioms of rationality. Preliminary discount rates pooled, as commonly found in such empirical investigations, indicated hyperbolic discount rates. As Nir (2004) stated, "*people are impatient at present, but claim to become more patient in the future*", which is typical of hyperbolic discounting. Although several authors (Laibson, 1997, Rubinstein, 2003) argue that hyperbolic discounting is time-inconsistent and therefore gives rise to many conceptual problems, we do expect ourselves to wiser and become more patient as we age. We therefore allowed hyperbolic time preferences to be just as rational as exponential time preferences.

We, however, have a systematic methodology towards exclusion factors with regards to time preference investigations through the Von Newman-Morgenstern rationality axioms (Von Neumann and Morgenstern, 1953). It is typical of preference investigations in the literature that a non-zero percentage of respondents make irrational choices. For example, Bobinac *et al* (2011) defined irrationality if an answer their open-ended question did not align more than once to the respondent's prior programme choices. They found that 7.4% were inconsistent. Given our small pilot sample size, the observed proportion was deemed adequate in order to pursue our web-based questionnaire for a more representative sample of the Dutch population.

4.a Appendix: Final questionnaire

Part A

This part of the questionnaire is on the individual's personal situation, work and income.

A1. What is your gender?

- (a) Female
- (b) Male

A2. What is your highest level of education? (High school and college levels are in Dutch terminology)

- (a) Primary school
- (b) High school: "huishoudschool, LTS, LEAO, VMBO, etc."
- (c) High school: "VMBO theoretische leerweg, MAVO, (M)ULO, MMS, three years of HBS"
- (d) High school: "HAVO, VWO, HBS"
- (e) College: "MTS, MEAO, etc."
- (f) University/College: Degree: BA, BS, Ing.
- (g) University/College: Degree MA, MS, Drs., Ir., PhD
- (h) Etcetera

Part B

Part B addresses the individuals quality of life.

The following questions are about your own state of health today. Please indicate which statements most accurately describe your own health today.

B1. Mobility

- (a) I don't have trouble walking
- (b) I do have some trouble walking
- (c) I am confined to bed

B2. Self-care

- (a) I don't have problems with washing or dressing myself
- (b) I have some problems with washing or dressing myself
- (c) I am unable to wash or dress myself

B3. Usual activities (e.g. work, study, household, family or leisure activities)

- (a) I have no problems with performing my usual activities
- (b) I have some problems with performing my usual activities
- (c) I am unable to perform my usual activities

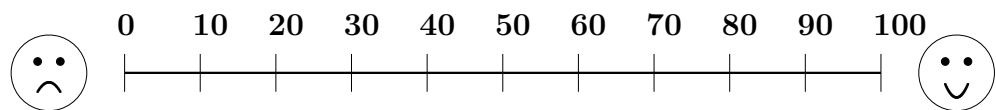
B4. Pain and/or discomforts

- (a) I don't have any pain or discomforts
- (b) I have moderate pain and/or discomforts
- (c) I have extreme pain and/or discomforts

B5. Mood

- (a) I am not anxious or depressed
- (b) I am moderately anxious and/or depressed
- (c) I am extremely anxious and/or depressed

B6. We would like you to rate the current state of your health, in your opinion, on the scale below. The best state you can imagine is marked 100 and the worst state you can imagine is marked 0.



Part C

This part investigates the time preferences, part C1 relates to time preferences for money and C2 relates to that for quality of life.

C1. Income

The following questions are on your preferences for receiving amounts of money. What would you choose?

Suppose that the value of money does not change over time (so no inflation occurs).

1. If you could choose between receiving €30.000.- one year from now or receiving €30.000.- five years from now, what would you choose?

- (a) €30.000.- one year from now
- (b) €30.000.- five years from now
- (c) Both options are equal to me

(exact numericals in following questions are determined by the software based on the previous answers)

2. And if you could choose between receiving €30.000.- in one year from now or receiving €75.000.- in five years from now, what would you choose?

- (a) €30.000.- in one year from now
- (b) €75.000.- in five years from now
- (c) Both options are equal to me

3. And if you could choose between receiving €30.000.- one year from now or receiving €55.000,- five years from now, what would you choose?

- (a) €30.000,- one year from now
- (b) €55.000,- five years from now
- (c) Both options are equal to me

4. And if you could choose between receiving €30.000.- one year from now or receiving €40.000,- five years from now, what would you choose?

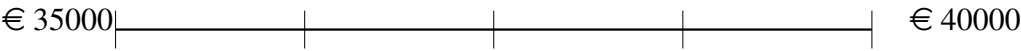
- (a) €30.000,- one year from now
- (b) €40.000,- five years from now
- (c) Both options are equal to me

5. And if you could choose between receiving €30.000.- one year from now or receiving €35.000,- five years from now, what would you choose?

- (a) €30.000,- one year from now
- (b) €35.000,- five years from now
- (c) Both options are equal to me

6. According to your answers, receiving an amount of €30.000,- *one* year from now has an equal value to you as receiving an amount of somewhere between €35.000,- and €40.000,- *five* years from now. (Range is been determined by the software, range shown is example)

We would like you to indicate on the scale below, the exact amount between €35.000,- and €40.000,-which, if received five years from now, would have the exact same value to you as receiving €30,000,- one year from now.

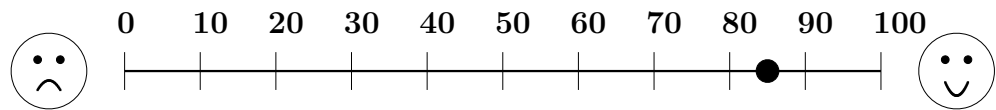


C2. Health

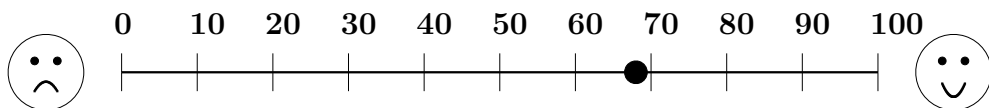
The following questions are on health and your preference. We would like you to choose from three options and mark which option you prefer.

First, we would like you to pay attention to the following states of health, just to get an idea of how it works. You don't have to choose yet.

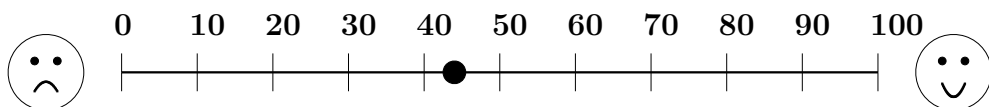
1. Decreasing mobility due to walking problems gives a 15 points loss.



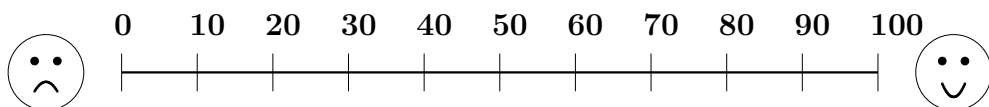
2. Severe anxiety/depression gives a 32 points loss.



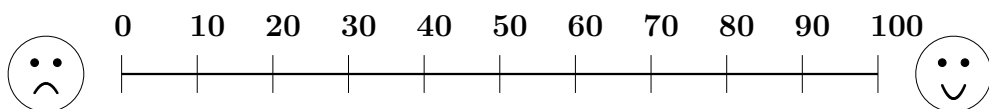
3. Severe pain and at the same time being unable to perform usual activities gives a 56 points loss.



On question B6, you indicated your own health with ... points (*score is shown*)



Suppose, that due to a disease you lost 30 points of your health (so 30 points compared to the estimation you gave at B6; *again score is shown*)



Attention: the loss of health occurs all of a sudden, you don't know beforehand when it will occur.

Assume, this lower health rate would last for a year, after that your health will return back to its normal state.

1. Suppose this temporal loss of health would occur one year, or five years from now. What would you prefer?

- (a) A loss of 30 points one year from now
- (b) A loss of 30 points five years from now
- (c) Both options are equal to me.

(exact numericals in following questions are determined by the software based on the previous answers)

2. Suppose the decrease in health would be larger when it occurs later, so smaller when it occurs earlier. What would your choice be in that case?

- (a) A loss of 25 points one year from now
- (b) A loss of 30 points five years from now
- (c) Both options are equal to me.

3. Suppose the decrease in health would be larger when it occurs later, so smaller when it occurs earlier. What would your choice be in that case?

- (a) A loss of 15 points one year from now
- (b) A loss of 30 points five years from now

(c) Both options are equal to me.

4. According to your answers, losing 30 points *five* years from now has an equal value to you as losing an amount of points of somewhere between 15 and 25 points in *one* year from now.

(Range is determined by the computer; range shown is example)

We would like you to indicate on the scale below, the exact number of points between 15 and 25, which, if lost one year from now, would have the exact same value to you as losing 30 points five years from now.

15 pts |-----|-----|-----|-----| 25 pts

Part D

Part D contains questions about future expectations in general.

D1. Question on living in good health

At the moment people in the Netherlands live on average 65 years in good health conditions. Please fill in the number of years you expect people in the Netherlands to live in good health in 5, 10, 20 and 40 years.

Number of years in good health now	65 years
Number of years in good health in 5 years
Number of years in good health in 10 years
Number of years in good health in 20 years
Number of years in good health in 40 years

D2. Question on life expectancy

At the moment people in the Netherlands, on average, reach the age of 79 years old. Please fill in the average age, you expect people in the Netherlands will reach 5, 10, 20 and 40 years from now.

Life expectancy now	79 years
Life expectancy in 5 years
Life expectancy in 10 years
Life expectancy in 20 years
Life expectancy in 40 years

D3. Question on income

At the moment the average income in the Netherlands is € 32.500,- per year. Please fill in the average income you expect to be earned in the Netherlands 5, 10, 20 and 40 years from now.

Average annual income now	€ 32.500
Average annual income in 5 years
Average annual income in 10 years
Average annual income in 20 years
Average annual income in 40 years

Thank you kindly for your cooperation

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Chapter 5

Empirical Evidence

¹ Draft questionnaires, having been piloted among a convenience sample of 1st year pharmacy students (Parouty *et al*, 2013), the pilot strengthened the initial idea on the appropriateness and validity of the question formulations and enabled us to pinpoint questions in any detail considered relevant. We are, thus, in a suitable position to assess whether rates of time preference for health outcomes are lower than those of money. Time preference investigations in health have, thus far, mostly involved life years gained and life saved. The Quality Adjusted Life Year (QALY) which is recommended by several bodies is however a multiplicative measure of life duration and quality of life. In particular, for the current purpose, we choose to find rates of time preference for money and for quality of life for the health part.

Apart from consistently comparing preferences for quality of life with those for money, we will also investigate how some population characteristics reflect in time preferences for both money and quality of life. More specifically, with this chapter, we aim to contribute to three discussions concerning discounting: (i) we want to

¹Based on

PAROUTY MBY, KROOSHOF DGM AND POSTMA MJ (2013) *On some endogenous determinants of time preferences for money and quality of life* submitted

provide further empirical evidence on the rationale for differential discounting, (ii) we want to validly assess the impact of certain population characteristics on time preferences and (iii) we aim to empirically elicit population time preferences for money. Sampling was done using a representative sample of the Dutch population and was managed by *Samplingsurvey International* (Rotterdam NL, www.surveysamplinginternational.com), a service company specialized and experienced in sampling. In particular, a sample of 847 individuals was questionnaired in 2011 and formed the basis of our analysis.

We sought to assess how individuals would compare gains of money with a year's delay from now to gains of money with delays of 5, 10, 20 and 40 years from now and gains in quality of life with a year's delay from now to gains in quality of life with the four other delays. We did not impose any exclusion criteria on the individuals since we wish to investigate the effect of certain population characteristics on time preferences and seek to gather maximum information. It is also important to note that we did not provide an immediate gain as part of any choices because we wished for the Neocortex and the Thalamus, in the individual's mind, to be equally activated while responding.

5.1 Methods

The respondents were asked to compare a gain of €30,000 one year from now, to a sum of money to which they would be indifferent to after the four different delays. With regards to quality of life, individuals were asked to imagine that 30% of their quality of life will be removed in one year, with 0% being the worst imaginable health state and 100% being the best imaginable health state. Compared to that utility loss in one year, individuals were asked to state the percentage to which they would be indifferent to after the different future delays. In order to obtain the discount rates for quality

of life, we assumed a perfect negative correlation between loss of health quality and gain in health quality; although it might be that our assumption needs further investigation.

Respondents were categorized into different mutually exclusive subgroups. Our first partitioning criterion was gender since males and females were suspected to have differing time preferences. Further, since individuals with poor health states might place higher importance to current health, etcetera, we also included questions on quality of life using EuroQol (EQ-5D) and visual analogue scale (VAS) to serve as a further partition basis (see Appendix). Partitioning was performed on the answers to EQ-5D and subsequent sorting was done on VAS if EQ5D equalled. Responses for individuals with relatively good health were compared to individuals with relatively poor health. These were termed high health status and low health status respectively.

A further partition of the sample involved education level. Literature has a huge repository regarding the diverging time preferences between relatively educated individuals and relatively uneducated individuals. Empirical evidences suggest that educated individuals are more patient than uneducated individuals (Oreopoulos, 2011). If the hypothesis that schooling increases patience is true, then we expect lower time preferences for the more educated individuals. We partitioned our sample into two groups; individuals who had Middelbaar beroepsonderwijs (MTS, MEAO, etc..), individuals with Hoger beroepsonderwijs and individuals with university education were classified in "higher education". The remaining was considered to have "lower education".

Next, we considered age which is commonly known as being a strong determinant for varying degrees of time preferences. The former has long been argued to correlate with optimistic/pessimistic views about the future). For example, Steinberg *et al* (2009) provided evidences that individuals younger than 16 years old express

higher preferences to accept a smaller reward sooner than a larger reward that is delayed. Younger people generally tend to have a relatively more optimistic view of their future. Furthermore, parental longevity has also been found to correlate to offspring's optimism (Rius-Ottenheim *et al*, 2012). It is generally argued that increased optimism is associated with higher discount rates (Berndsen and van der Pligt, 2001). In our inferences, direct measurement of beliefs was carried out. Responses were classified into two mutually exclusive sets; namely "optimistic" and "pessimistic" about the future. We asked respondents about their general feelings regarding the future, in particular whether one is basically positively or negatively inclined in this respect (optimistic or pessimistic) with regards to quality of life, life expectancy and income (See part D in Appendix). Individuals having all responses, at delays of 20 years, that were lower than the number stated as being the case right now, were deemed pessimistic. The remaining was classified as optimistic. Discount rates were pooled at each individual time point by the formula:

$$Q_{t_2} = Q_{t_1}(1 + r_Q)^{t_2 - t_1}$$

with:

r_Q : annual discount rate for outcome Q

Q_{t_1} : amount t_1 years from now

Q_{t_2} : amount t_2 years from now

Notably, Q 's can reflect money or quality of life, corresponding with the questionnaires. From the individual discount rates, the means were calculated and were plotted against the time of delay. Since this chapter aims to investigate the differences in time preferences rather than expressing exact numericals, linear regression, as previously done by Cropper *et al* (1992), was fitted to each sub-category. Linear regression provides a useful way of investigating the differences, especially, since the slopes for the subgroups are roughly expected to be similar to that of the overall sample. As a result, the intercepts

should be numerical markers of the differences in time preferences with respect to differing sub-groups.

5.2 Results

Table 5.1 below represents the sample properties for the 847 individuals. As obviously not all questions were answered consistently by all eligible individuals, denominators for sub-analyses – for example, on optimistic/pessimistic future perspectives – may differ.

Table 5.1: Partitioning of overall sample set in mutually exclusive subsets

Respondent Characteristics	N=847 (100%)
Gender	
Female	422 (50%)
Male	425 (50%)
Health state	
Higher health status	424 (50%)
Lower health status	423 (50%)
Optimistic on	
3 questions	377 (45%)
2 questions	165 (19%)
1 question	146 (17%)
0 question	159 (19%)
Education	
Higher education	491 (58%)
Lower education	356 (42%)

Figure 5.1 shows the overall discount rates, suggesting declining discount rates over increasing time of delay, in contrast to a constant rate. The annual discount rate for money consistently lies above the annual discount rate of quality of life, with that for money being at least twice that for health over the whole spectrum. Notably, this factor increases to at least eight times with increasing time of delay.

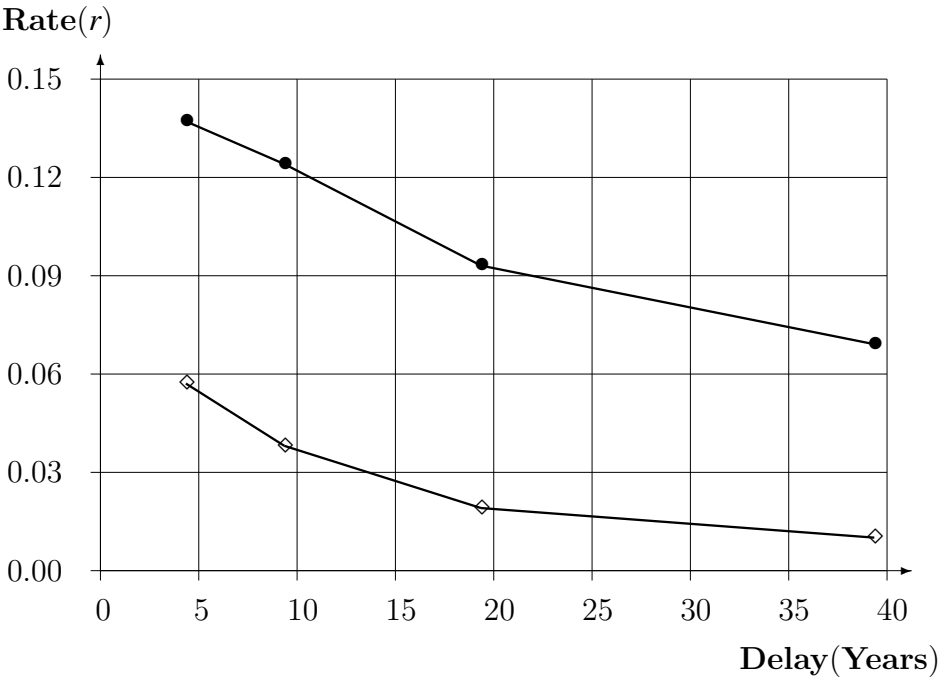


Figure 5.1: Annual discount rate for money(●) and health(◇) plotted against time of delay.

Figure 5.2 shows results for men and women on money, again, indicating a decline over increasing time of delay. The annual discount rate of female respondents lays approximately 13% above the annual discount rate of male respondents for sooner delays, decreasing to about 7% with increasing delays.

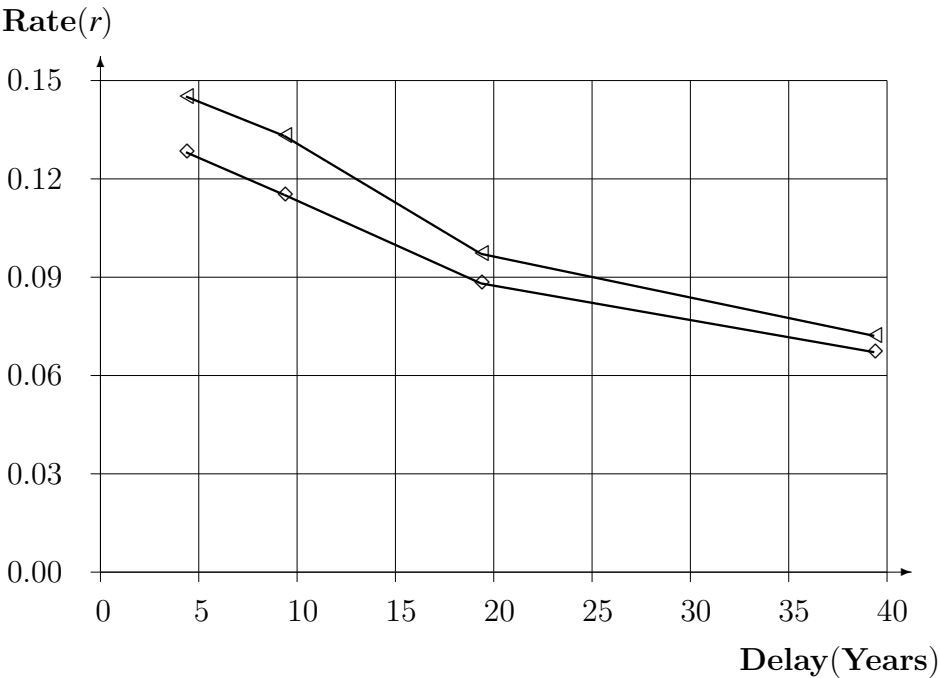


Figure 5.2: Annual discount rate for money plotted against time of delay. Female(\triangle) respondents are compared to male(\diamond) respondents.

Figure 5.3 illustrates –for money– the typical profile for optimistic versus pessimistic respondents. In particular, the tendency shows that optimistic individuals tend to elicit relatively higher discount rates, although the profiles seem to regress to a mean at around a delay of five years from the present. This is in line with other results, where ad extremum individuals with a very pessimistic view on the future could even start exhibiting negative discount rates. This was however not seen in our data.

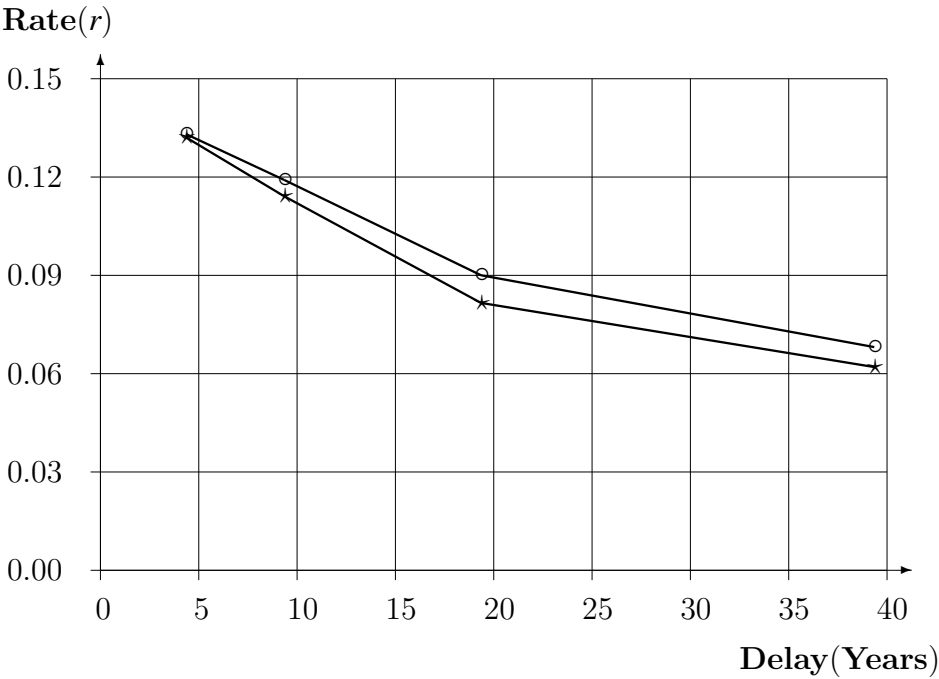


Figure 5.3: Annual discount rate for money plotted against time of delay. Respondents who are optimistic (\circ) are compared to pessimistic (\star) respondents.

Figure 5.4 illustrates the results after sorting on health status; i.e. on EQ5D and VAS. The figure shows that individuals in the lower 50%-percentile of EQ5D/VAS begin to discount at approximately similar rates to those in the upper percentile and discount up to 102 % higher than those in the upper percentile within the furthest delay.

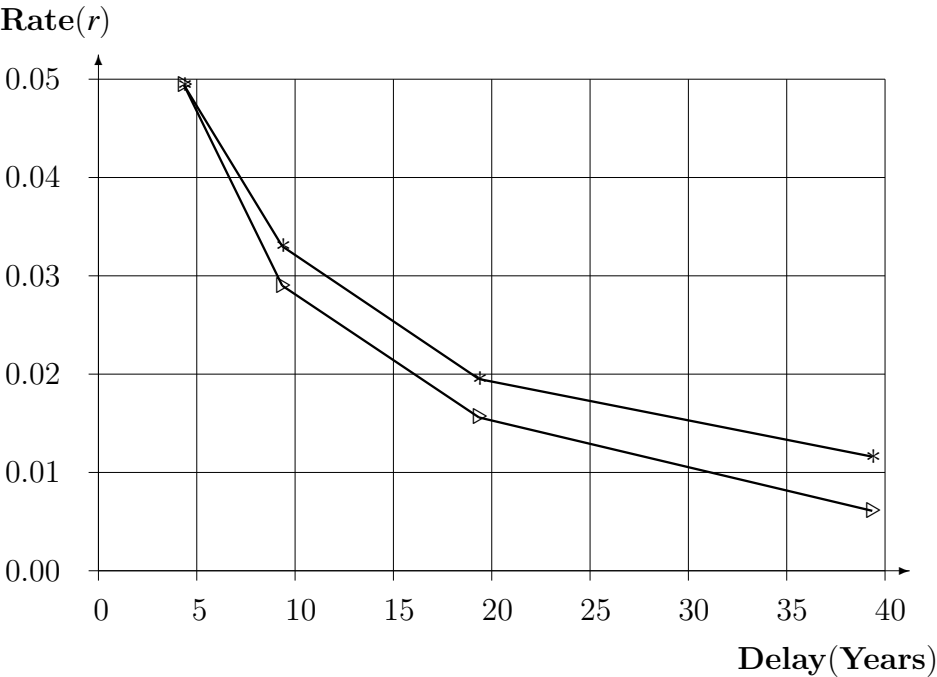


Figure 5.4: Annual discount rate for health plotted against time of delay. Respondents belonging to the upper 50% regarding health status (\triangleright) are compared to those belonging to the lower 50% regarding health status (*).

Table 5.2: Linear regression fits on the discount rate for money ($r = \alpha + \beta t$) and 95% confidence intervals, *C.I.*'s

Characteristic	α (95% <i>C.I.</i>)	β (95% <i>C.I.</i>)
Overall	0.132 (0.098, 0.167)	-0.002 (-0.003, -0.000)
Male	0.127 (0.091, 0.163)	-0.002 (-0.003, -0.000)
Female	0.138 (0.104, 0.172)	-0.002 (-0.003, -0.000)
Higher health status	0.126 (0.095, 0.157)	-0.002 (-0.003, -0.000)
Lower health status	0.139 (0.102, 0.177)	-0.002 (-0.004, -0.000)
Optimistic	0.133 (0.101, 0.165)	-0.002 (-0.003, -0.000)
Pessimistic	0.130 (0.087, 0.173)	-0.002 (-0.004, -0.000)
Higher Education	0.123 (0.098, 0.148)	-0.002 (-0.003, -0.000)
Lower Education	0.146 (0.094, 0.168)	-0.002 (-0.005, -0.000)

The decline in the discount rate for money was approximately the same in all cases. As such, the regression analysis confirmed what was already suspected; males, individuals with higher quality of life, pessimistic individuals and individuals with relatively higher education have a lower discount rate than their counterparts; as observed from the intercept, α . Next, the regressions for the rate of time preference for quality of life are shown in the table below.

Table 5.3: Linear regression fits on the discount rate for quality of life ($r = \alpha + \beta t$) and 95% confidence intervals, *C.I.*'s

Characteristic	α (95% <i>C.I.</i>)	β (95% <i>C.I.</i>)
Overall	0.044 (0.010, 0.079)	-0.001 (-0.003, 0.001)
Male	0.042 (0.008, 0.076)	-0.001 (-0.002, 0.001)
Female	0.048 (0.011, 0.085)	-0.001 (-0.003, 0.001)
Higher health status	0.044 (0.007, 0.082)	-0.001 (-0.003, 0.001)
Lower health status	0.045 (0.013, 0.078)	-0.001 (-0.002, 0.000)
Optimistic	0.052 (0.012, 0.093)	-0.001 (-0.003, 0.001)
Pessimistic	0.014 (-0.007, 0.034)	0.000 (-0.001, 0.001)
Higher Education	0.048 (0.011, 0.085)	-0.001 (-0.003, 0.000)
Lower education	0.041 (0.008, 0.073)	-0.001 (-0.002, 0.001)

Again, the same trends were observed for quality of life as for money. Further, the discount rates for both money and health decrease over increasing time of delay as indicated by the correlation coefficient of almost negative one, $\rho_{t,r_M} = -0.964$ and $\rho_{t,r_Q} = -0.896$; where ρ 's denote the sample correlation coefficients for money and for quality of life, respectively. Linear regressions for money and health versus delay indicated that the discount rate function for money is higher than that of health for **all** delays. Linear regressions on individuals with different health statuses also indicated higher health status individuals correspond to lower discount rates. Similar discount rate functions were estimated for subgroups on gender and views about the future. Optimistic individuals and females elicit higher discount rates.

While observations were all in line with literature with regards to discounting money, relatively more educated individuals had a higher time preference for quality of life in contrast to a lower time preference for money when compared to relatively less educated individuals. It is often argued that educated individuals are more patient, which explains the lower time preference for money. However, as Becker and Mulligan (1997) argue, schooling trains the imagination of future rewards and better foresight. As such, in their own words, *“educated people should be more productive at reducing the remoteness of future pleasures”*. Thus, the ubiquitous expectation of income to increase as we age but that of quality of life to decrease provides a plausible explanation. In Fischer's words, it appears that relatively higher educated individuals discount future quality of life at a high rate due to *“the thought that provision for the present is necessary both for the present itself and for the future as well”* (Fischer, 1930).

Differences in discount rates related to grouping of our sample with respect to gender, health status and general attitude (pessimistic/optimistic) were sometimes consistent across time, attributes and types of analysis. Despite being interesting per se, such differences might not

trigger any impacts on recommendations on discounting as, generally, for that purpose, the societal perspective is dominant. Notably, within society the heterogeneity over gender, attitudes and health status is implicit and therefore the recommended societal perspective should entangle the preferences of these subgroups in an integrated manner, rather than by the constituting elements of society.

5.3 Discussion

This chapter obviously has some limitations; the most important of which remains inferring broader societal perspective in line with cost-effectiveness analyses from an empirical research on discounting. We have designed the questions as specifically and objectively as possible and refrained from any potential hints for directions of answers, however always seeking the alignment with broader perspectives takes in health-economic analyses from societal perspective *ad ultimo*. Further work remains to be done here; we however remark that the qualitative differences and trends observed in our study should be the focal point rather than the exact quantitative results.

Nonetheless, summarizing our findings:

1. The discount rate for money is consistently higher than that of health, for all horizons
2. The discount rate decreases over increasing time of delay
3. The rate of decline of the discount rate decreases over increasing time of delay
4. A lower health status corresponds with a higher discount rate
5. Females elicit higher discount rates compared to males
6. Optimistic individuals elicit higher discount rates than pessimistic individuals

7. Relatively more educated individuals have lower time preferences for money but higher time preferences for health compared to relatively uneducated individuals.

It is well substantiated in literature that empirically found discount rates are relatively higher than numericals featuring in various (inter-)national guidelines for pharmacoeconomic or health economic purposes (ISPOR website, 2012). Having that in mind, we rather draw inferences from the relative results on discount rates of, for example, health versus money and males versus females, optimistic versus pessimistic and on decreasing trends in time rather than focusing on exact levels of discounting. Decreasing trends in the discount rates as found here are in line with suggestions made in the literature and some guidelines (Bazelon, 2002, Beutels *et al*, 2008, UK Treasury, 2013, WHO Guidelines, 2012). Yet, in application, we do not often notice decreasing discount rates being used. However one might expect the uptake of decreasing rates; in particular in the area of long-term prevention such as vaccination programs. The methodology has been investigated in this specific area, taking the case of HPV vaccination, for example, in chapter 2.

Our results, however, support differential discounting, particularly, given our sample was representative of the Dutch population. The discussion on differential discounting has a history dating back at least 2 decennia (Parsonage and Neuberger, 1992). Countries for which the discussion has actually impacted on the discount rates in the guidelines are the UK, the Netherlands and Belgium so far. The ongoing discussion in the UK is reflected in a recent UK-initiative (NICE Report, 2012) that recommends NICE to allow differential discounting of costs and health effects if interventions concern investment upfront and benefits accruing over future delays; i.e. for interventions in children, prevention and public health (HPV-vaccination would indeed fit in all categories). Furthermore, the report refers again to the UK Treasury (2013) that already suggested decreasing discounting with increasing delays, a while ago. Both recommenda-

tions are in line with our (and others') empirical findings.

However, with respect to our argument in chapter 3, the recent NICE Report (2012) summarizes that societal time preference can be considered to reflect pure time preference, uncertainty and growth (in the economy in particular). The latter reduces the marginal utility of additional units. Uncertainty is implicit in all aspects of any decision, both money and health related. For example, uncertainty in the economic analysis of the HPV-vaccines may relate to the fact that prevention of cervical cancers pertain to periods decennia after actual vaccination; i.e. it is uncertain whether in the meantime an effective pharmacotherapy for this cancer might be developed or not. This uncertainty clearly impacts on economic aspects (savings on cancer costs) and health aspects (death and quality losses due to cancer).

A discount rate of 1.5% might nonetheless adequately reflect this part of time preference for a broad range of developed economies, including the UK and the Netherlands. Economic growth gives an additional motivation for an overall higher time preference for money, often chosen in the range 3-5% (Gold *et al*, 1996, Gravelle and Smith, 2001, ISPOR website, 2012, Klok *et al*, 2005, NICE Report, 2012). Relatively more modest growth in health (for example, in life expectancy as we list in chapter 3) has motivated arguments about a lower discount rate for health, challenging the invariance assumption in the discounted utility (*DU*) model. Our empirical analysis, however, reinforces the argument for differential discounting. Perhaps, when considering health outcomes, individuals do not exhibit *pure* impatience. In any case, based on a representative sample of the Dutch population, a lower rate for health is strongly motivated.

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Part II

A Framework for Differential Discounting

Chapter 6

Differential Discounting

¹ Empirical investigations, provided in the previous chapter, suggest that the discount rates of health outcomes are consistently lower than those of money, especially given that chapter 4 ensured that question formulations for both were, as far as possible, similar. Furthermore, chapter 3 argues that the use of a single rate is not theoretically defensible with regards to discounting health. None of these are as important as the implications provided in chapter 2. The cost-effectiveness of vaccination for human papilloma virus (HPV), where savings on cancer costs, averted mortality and QALY gains are all decennia after teenage HPV-vaccination programs, clearly indicates that discounting at various rates ranging up to 4% may reduce savings and QALYs up to 5-fold (Brisson *et al*, 2007, Westra *et al*, 2012). Grossly this 5-fold reduction would be mirrored by a 5-fold increase in the cost-effectiveness ratio of HPV-vaccination if the discounted ratio would be compared with the undiscounted ratio.

¹Based on

POSTMA MJ, PAROUTY MBY AND WESTRA TA (2013) *Accumulating evidence for the case of differential discounting* Expert Rev. Clin. Pharmacol. 6(1), pg 1-3

WESTRA TA, PAROUTY MBY, WILSCHUT JC, BOERSMA C AND POSTMA MJ (2011) *Re:Practical Implications of Differential Discounting of Costs and Health Effects in Cost-Effectiveness Analysis* Value in Health 14, pg 1173-1175

The lack of a theoretical underpinning in the attempts to bring outcomes in harmony with ethics begs the question about whether government officials should reflect our instincts in policies regarding discounting of health, even when these are irrational. An ethically defensible guideline should, however, also be theoretically defensible. Arguably, policies should incorporate a democratic public agent's time preference in society, reflecting differences in preferences due to timing of outcomes, be them related to monetary outcomes (costs and savings), health (life years and quality of life) or other consumption goods. With that regards, several bodies, including NICE (2008), prescribe a single rate for both costs and outcomes. The rationale for equal discounting of health gains dates back to the *postponement paradox* of Keeler and Cretin (1983).

They argued that discounting costs and effects at different rates would result in inconsistencies in decision making as it would then matter whether one would discount future benefits to compare them with current costs, or compound forward in time current costs to compare them with future benefits. The paradox of Keeler and Cretin (1983) implies that it would always be more cost-effective to wait until implementation of an intervention. The inappropriateness of this *waiting* strategy stems from a general economic paradox which Koopmans (1967) called: "*the paradox of the indefinitely postponed splurge*". A major economic assumption for this paradox is that the future return to capital is positive (as we have seen in Ramsey's derivation in chapter 3), and utility is increasing in consumption. Thus, not discounting benefits at the social discount rate would cause distant generations to be, in the words of Nozick (1974), *utility monsters*, due to excessive sacrifice of the current generation (Arrow, 1983, Chakravarty, 1962, Solow, 1974).

When Keeler and Cretin (1983) "*reinvented* this paradox to prove that future health benefits must be discounted at the same rate as costs" (Frederick, 1999), the inconsistencies of differential discount-

ing were deemed inappropriate. However, years after Keeler and Cretin (1983) transferred this paradox to health outcomes, Gravelle and Smith (2001) contended their *proof* with a simple argument that the postponement paradox has a fundamental assumption that the monetary value of health is constant into the indefinite future. While constancy of the monetary value of health in time is a possibility, it was not deemed to be a well-founded assumption. The last decade has thus seen adoption of differential rates in a few guidelines.

Primarily based on the work of Gravelle and Smith (2001), the UK was prominent to be the first country to adopt differential discounting around the turn of the century with discount rates of 6% for money and 1.5% for health. However, in 2004, the UK changed back to equal discounting after reiterating the landmark paper by Keeler and Cretin. During that same time, Klok *et al* (2005) adapted the Gravelle and Smith model to the Netherlands, estimating a 4% discount rate for money and 1.5% for health that subsequently led to a diversion from equal to differential discounting in the Dutch guidelines. The opposite switches in the UK and the Netherlands led to an interesting polemic between authors involved in NICE and in the Dutch guidelines (Brouwer *et al*, 2005, Claxton *et al*, 2006). Belgium followed the Dutch approach just a few years later (KCE, 2013). Notably, the two “low countries” are the only ones in the world prescribing differential discounting for a human life given its specificities (CvZ, 2013, KCE, 2013).

6.1 Discounting dynamism

The non-constancy of the growing value of health through time calls attention to multifarious regards to cost-effectiveness analyses (CEA). Since in an equilibrium state, augmentation somewhere should mirror in a reduction some other where, the complex considerations entailing differential discounting have refrained economists from delving beyond the statement that “*the monetary value of health might*

grow over time". Although health has often been assumed to have a one-to-one relationship with wealth, it seems that a situation of abundance of wealth need not necessarily imply abundance of health and vice versa; though the converse is even more true.

While economists might argue that the industrial sector is associated with improving both wealth and health, an idea that has been recently explored by Pritchett and Summers (1996) in their paper entitled "Wealthier is healthier", historians such as Szreter (2004) noted that the paradox is that industrialization itself, like all forms of economic growth, exerts intrinsically negative population health effects among the communities that are most directly involved in the transformations which it entails. He also noted that this proposition grows even stronger when it is realized that in majority of the industrialization cases of today's successful developed economies, their historical demographic trends exhibit the same pattern of a negative inflection in health trends during the decades in which industrialization mostly affected their populations.

When we regard the timely demand-supply of some measure of health, it seems that the correlation between health effects and income through time is not strictly one. Rather, the market for health states is a dynamic one. For example, with changes in preferences for health states, measures such as the EuroQol-5D for a societal valuation or the EQ VAS for an individual valuation, are also affected. We have already seen, in the previous chapter, that individuals with a higher health state discount at a different rate than those with a relatively lower health state. Thus, health state measures, in turn, alter our utility and welfare valuations for health. And consequently, this would impact on a different rate of time preference for health. And the new discount rate, in turn, changes the list of cost-effective interventions and the whole cycle begins over again.

Different sectors of the economy impact on our health and health states preferences and these preferences, in turn, impact on the dis-

count factor. Following this line of thought, it seems that the whole economic system has to be taken into account when attempting to value the future. Our former concept of a linear sequence of events from cause to effect seems to forsake the interlinkage that exists among different sectors of the economy and the models used this far have been those of mere *equations* which traditionally involve two quantities. Simply transferring discounting principles from money to health, not only lacks the theoretical foundations, but also has dramatic consequences on the rights of future generations.

As a result, since there seems to be, firstly, an ethical need, secondly, a theoretical motivation, and thirdly, empirical evidences for a differential rate for health, it would appear that there is need to review our theoretical framework regarding discounting in general. General health economic literature assumes an economy composed of health and income only; and maximization problems, whether concerning the discounted health outcomes or the present consumption value of health, are often subject to an exogenous budget constraint (Claxton *et al*, 2011). I, however, argue that growth of health effects depends, not only on wealth, but also, on growth of other sectors (commodities) of the economy which are directly or indirectly related to health.

6.2 Dynamics denotation

While non-constancy of a monetary or some other measurable *value* of health through time suggests extending the invariant discount rate assumption to commodity-specific and time-specific ones, our imperfect foresight (and inability to (for-)see all the levers and pulleys behind the stage) suggests the use of measure theoretic probability. The impact of a lack of such unification between imperfect foresight and a potentially growing value of health has recently gained our attention with regards to multi-cohort CEA specifications.

O'Mahony *et al* (2011) show that the incremental cost-effectiveness ratio (ICER) for HPV vaccination decreases if differential discounting is applied with an increasing number of annual vaccinated cohorts analyzed; that is, going from single cohort, through 10 and 20, to 30 cohorts. In their wordings, they label the choice of how many cohorts to include in the model as arbitrary variation in study specification and subsequently the related outcomes as arbitrary variation in results. With differential discounting, the ICER decreases in their hypothetical calculations from almost €30,000, through €27,000 (10 cohorts) and €24,000 (20 cohorts), to €22,000 per quality-adjusted life-year if 30 cohorts are analyzed.

We, however, argue that the choice of the number of generations to include in the model is far from arbitrary, as is the corresponding outcome. In particular, the number of cohorts included in the model should directly reflect the envisioned time horizon for the implementation of the vaccination. As generally a vaccination program is not foreseen for 1 year only, inclusion of multicohorts is an adequate approach. The exact number of cohorts is to be discussed; however, rather than being arbitrary it should reflect a valid idea on the minimum period for the vaccination program to be in place. That many researchers do choose a single-cohort model for analysis can easily be motivated by the fact that

1. a model as least complex as possible should always be strived for.
2. this would always represent a conservative approach, given the downward slope of the ICER as a function of the number of cohorts included.

If specification of the time horizon for intervention upfront is not desired or possible in a multicohort model, the following reasoning might be considered. Notably, the authors consistently discount costs and effects to the first year; that is, the year in which the first cohort is vaccinated. It might, however, be argued that the benefits

for a particular cohort should be discounted to the moment the intervention took place in this specific cohort, which would result in identical ICERs for each subsequent cohort.

Correspondingly and consistently based on the theoretical basis of differential discounting, it might be argued that the willingness-to-pay threshold increases for subsequent cohorts. In particular, because of the increasing value of health over time, the willingness-to-pay threshold increases, rendering the constant ICERs more cost-effective year by year. We argue that in judging the ICERs of subsequent cohorts one should take this change in the willingness-to-pay threshold into account. So, potentially, it is possible that because of the increasing value of health, an intervention might not be cost-effective for the current cohort, while the intervention will become cost-effective for future cohorts. While this is fully in line with the rationale behind differential discounting, it appears that probabilistic statements about future ought to be feasible in order to admit differential rates with caution.

6.3 Denouement

Hence, proxies for health that policy makers should consider ought to be all those factors that affect growth in health such as education, social cost of carbon, to name a few. Nobel laureate Myrdal (1963) also argued that production is a circular and cumulative causation process, implying that income growth alone cannot fully reflect growth in health. I therefore argue that we should revise our current theoretical approach towards discounting such that our assumptions are better representatives of our economic system. Furthermore, it seems that, in order to account for the inter-dependence of the different sectors of the economy, there is need to go beyond a 2-dimensional health/wealth model.

Notably, models for vaccination, in general, should consider herd-

immunity benefits through the indirect protection of nonvaccinated individuals, and the size of this effect will differ between different vaccinated cohorts. In such a dynamic model, consequently, the ICERs will change from year to year even under equal discounting since herd-immunity is dependent on the changing proportion of vaccinated individuals. So, besides changes in cost-effectiveness over various cohorts due to discounting, crucial changes in cost-effectiveness will become apparent because of crucial changes in vaccination coverage and epidemiology that quickly overrule variations due to discounting. In conclusion, I suggest the consideration of an n -commodity intertemporal economy with the variability of intertemporal indifferences being captured by a measure-theoretic probability distribution.

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Chapter 7

A Relaxation of Invariance

¹ Given the lack of theoretical defensibility of an equal discount rate for costs and health outcomes argued, this chapter relaxes the assumption of a commodity-invariant discount rate. Moreover, it is often empirically remarked that individuals do not coordinate their intertemporal preferences with pricing choices (Amir and Ariely, 2007, Frederick and Loewenstein, 2008, Kahneman and Varey, 1991, Loewenstein and Prelec, 1993). Frederick and Loewenstein (2008), among others, found that intertemporal preferences generally do not have the expected mapping properties on intertemporal willingness-to-pay; i.e. higher (intertemporal) preferences do not generally reflect higher willingness-to-pay. As a result, time preferences for a certain commodity and for income, say, generally do not allow for inferences on time preferences for the marginal value of that commodity in terms of income.

Consequently, I assume a set of n -commodity-specific discount functions and provide a matrix-vector representation of marginal substitutions based on a model consistent expectation. Since we are

¹Based on

PAROUTY MBY, VISSER S AND POSTMA MJ (2013) *A consistent relaxation of the consumption invariant rate in the discounted-utility model* Theoretical Economic Letters, accepted

concerned with marginals, one need not necessarily specify the “absolute” functions, *per se*, but rather the relative change of a function compared to another. My model, being valid in cases of negative discount rates as well, derives from a more general concept. Our conceptualisation gained popularity, mostly among physicists, from Einstein’s general theory of relativity in which he considered a set of coordinate system where the metric tensor defined the type of space, flat or curved etcetera. In our case, we note that some function of well-being across time for an n -commodity economy can be geometrically represented by a function of the n -dimensional commodity space where each coordinate changes with time according to some specific function. With regards to the coordinate transformations, I shall assume that production processes of n different commodities are equivalent to some value-gaining processes such as Rae’s instruments (Rae, 1834), where the value gaining processes are, possibly, dissimilar. Thus a single point can be infinitely characterised by alternative time-dependent coordinate systems. Section 7.1 provides a theorem of the representation which I prove by induction.

Using arguments such as consistency in intertemporal choices and intercommodity wise, we then have a cyclical representation of marginal valuations. Cyclical mechanisms describing the economy have, for the past few decades, gained the attention of several economists (Leontief, 1986, Sraffa, 1960). The simplicity of the mathematics of input-output systems has led to extending such systems to open ones as well as to intertemporal ones by adding an additional growth term. An important facet in input-output systems generally (if not always) includes some arguments of equivalence relation. For example, Sraffa’s system equates the value (price times quantity) of a product of each industry to the value of all goods and services absorbed by this same industry². In our case, we first equate commodity i ’s current input quantities to it’s future quantity specified by some growth function. Although the quantities of commodities vary stochastically, as a first

²See appendix for a brief description of Sraffa’s system

approach, I propose a model-consistent expectation that assumes that commodities evolve along deterministic (expected) functions of time³. Section 7.2 provides a visual derivation based on some uniform measure of the commodities or purely on physical quantities of the commodities.

Our representation however allows for differential discounting which is a major debate in health economics. I shall mainly exemplify our representation by assuming that health is a commodity⁴. Furthermore, the representation also enables one to incorporate externalities in cost-effectiveness analyses. As such one might want to investigate the cost-effectiveness with regards to net-monetary gains as well as increased life expectancy or increased standards of life, say. Given that some economic transactions are known to increase income but decrease quality of life or life expectancy, say, I suggest that policy makers should account for externalities. Section 7.3 introduces illustrations with such marginal valuations and section 7.4 concludes.

7.1 A covariance representation of coordinate transformations

Suppose that we have an n -dimensional function, say $w(\mathbf{q})$ described by two sets of coordinate systems, say $(q_1^0, q_2^0, \dots, q_n^0)$ and $(q_1^T, q_2^T, \dots, q_n^T)$ where the coordinate transformation is given by $q_i^T = f(q_i^0, T)$ with $i = 1 \dots n$, say. Since a differential, dw , is uniquely characterised in the q^0 -frame of reference as well as uniquely characterised in the q^T -frame of reference, one could consider a differential, $d\mathbf{q}^0 \leftrightarrow dq_1^0, dq_2^0, \dots, dq_n^0$ which can also be characterised by $dq_1^T, dq_2^T, \dots, dq_n^T$. Assuming that we know the set of partial

³Throughout this chapter, subscripts shall denote commodities and superscripts shall denote time.

⁴We do not consider health as a commodity but shall however assume the latter for representative purposes given current debates that differential discounting might be warranted for Health Technology Assessments.

derivatives in the q^0 -frame of reference, $\partial q_i^0 / \partial q_j^0$, $i, j = 1 \dots n$, as well as in the q^T -frame of reference, $\partial q_i^T / \partial q_j^T$, $i, j = 1 \dots n$, we can form a matrix with those known components such that the eigenvector of that matrix, corresponding to an eigenvalue 1, represents the bijective coordinate transformation components, $\frac{\partial q_i^T}{\partial q_j^0}$, $i = 1 \dots n$. We then have a matrix-vector representation of partial derivatives since we are simply concerned with the derivative of an axis with respect to another (not necessarily orthogonal here).

Theorem 7.1.1. *Suppose that we have an n -dimensional space characterized by 2 sets of n -dimensional coordinates, q^0 and q^T , then, defining v_{ij}^t by $\frac{\partial q_j^t}{\partial q_i^t}$, the matrix $\mathbf{X} = (x_{ij})$ $i, j = 1..n$ given by,*

$$1) x_{ij} = v_{ij}^0 \text{ if } i < j$$

$$2) x_{ij} = v_{ji}^T \text{ if } i > j$$

$$3) x_{ij} = 1 - \sum_{i=1}^{j-1} v_{ij}^0 - \sum_{j=j+1}^n v_{ij}^T \text{ if } i = j$$

has an eigenvalue of 1 that corresponds to the eigenvector⁵ $\mathbf{e} = (\frac{\partial q_1^T}{\partial q_1^0}, \frac{\partial q_2^T}{\partial q_2^0}, \dots, \frac{\partial q_n^T}{\partial q_n^0})$

That is

$$\begin{pmatrix} 1 - \sum_{j=2}^n v_{1j}^T & v_{12}^0 & \dots & v_{1n}^0 \\ v_{12}^T & 1 - \sum_{i=1}^1 v_{i2}^0 - \sum_{j=3}^n v_{2j}^T & \dots & v_{2n}^0 \\ \vdots & \vdots & \ddots & \vdots \\ v_{1n}^T & v_{2n}^T & \dots & 1 - \sum_{i=1}^{n-1} v_{in}^0 \end{pmatrix} \begin{pmatrix} \frac{\partial q_1^T}{\partial q_1^0} \\ \frac{\partial q_2^T}{\partial q_2^0} \\ \vdots \\ \frac{\partial q_n^T}{\partial q_n^0} \end{pmatrix}$$

⁵The vector might, as well, be representative of scalars, such as a tensor of rank 0.

$$= \begin{pmatrix} \frac{\partial q_1^T}{\partial q_1^0} \\ \frac{\partial q_2^T}{\partial q_2^0} \\ \vdots \\ \frac{\partial q_n^T}{\partial q_n^0} \end{pmatrix}$$

Proof. Suppose there exists a matrix, \mathbf{X} , that is non-zero and non-diagonal, given by

$$1) x_{ij} = v_{ij}^0 \text{ if } i < j$$

$$2) x_{ij} = v_{ji}^T \text{ if } i > j$$

$$3) x_{ij} = 1 - \sum_{i=1}^{j-1} v_{ij}^0 - \sum_{j=j+1}^n v_{ij}^T \text{ if } i = j$$

such $\mathbf{X}\mathbf{e}' = \mathbf{e}'$ where $\mathbf{e} = (f(q_1^T), f(q_2^T), \dots, f(q_n^T))$ with $f(q_j^T) = \partial q_j^T / \partial q_j^0$ and $v_{ij}^t = \frac{\partial q_j^t}{\partial q_i^t}$, $i, j = 1 \dots n$, $t = 0, T$.

Then, $f(q_1^T)$ satisfies $\sum_{j=1}^n x_{1j} f(q_j^T) = f(q_1^T)$. i.e.

$$\begin{aligned} \sum_{j=1}^n x_{1j} f(q_j^T) &= \left(1 - \sum_{j=2}^n v_{1j}^T\right) f(q_1^T) + v_{12}^0 f(q_2^T) + \dots + v_{1n}^0 f(q_n^T) \\ &= \frac{\partial q_1^T}{\partial q_1^0} - \sum_{j=2}^n \frac{\partial q_j^T}{\partial q_1^T} \frac{\partial q_1^T}{\partial q_1^0} + \left(\frac{\partial q_2^0}{\partial q_1^0}\right) f(q_2^T) + \dots + \left(\frac{\partial q_n^0}{\partial q_1^0}\right) f(q_n^T) \\ &= \frac{\partial q_1^T}{\partial q_1^0} = f(q_1^T) \end{aligned}$$

Assume that $\sum_{j=1}^n x_{(i-1)j} f(q_j^T) = f(q_{(i-1)}^T)$ is true.

Then $\sum_{j=1}^n x_{ij} f(q_j^T)$, given by

$$\sum_{j=1}^n x_{ij} f(q_j^T) = \sum_{j=1}^{i-1} x_{ij} f(q_j^T) + x_{ii} f(q_i^T) + \sum_{j=i+1}^n x_{ij} f(q_j^T)$$

$$\begin{aligned}
&= \sum_{j=1}^{i-1} v_{ji}^T f(q_j^T) + \left(1 - \sum_{i=1}^{j-1} v_{ij}^0 - \sum_{j=j+1}^n v_{ij}^T\right) f(q_i^T) + \sum_{j=i+1}^n v_{ij}^0 f(q_j^T) \\
&= \sum_{j=1}^{i-1} \frac{\partial q_i^T}{\partial q_j^T} \frac{\partial q_j^T}{\partial q_j^0} + \left(1 - \sum_{i=1}^{j-1} \frac{\partial q_j^0}{\partial q_i^0} - \sum_{j=j+1}^n \frac{\partial q_j^T}{\partial q_i^T}\right) \frac{\partial q_i^T}{\partial q_i^0} + \sum_{j=i+1}^n \frac{\partial q_j^0}{\partial q_i^0} \frac{\partial q_j^T}{\partial q_j^0} \\
&= \sum_{j=1}^{i-1} \frac{\partial q_i^T}{\partial q_j^0} + \left(1 - \sum_{j=1}^{i-1} \frac{\partial q_i^0}{\partial q_j^0} - \sum_{j=j+1}^n \frac{\partial q_j^T}{\partial q_i^T}\right) \frac{\partial q_i^T}{\partial q_i^0} + \sum_{j=i+1}^n \frac{\partial q_j^T}{\partial q_i^0} \\
&= \sum_{j=1}^{i-1} \frac{\partial q_i^T}{\partial q_j^0} + \left(\frac{\partial q_i^T}{\partial q_i^0} - \sum_{j=1}^{i-1} \frac{\partial q_i^T}{\partial q_j^0} - \sum_{j=j+1}^n \frac{\partial q_j^T}{\partial q_i^0}\right) + \sum_{j=i+1}^n \frac{\partial q_j^T}{\partial q_i^0} \\
&= \frac{\partial q_i^T}{\partial q_i^0} = f(q_i^T)
\end{aligned}$$

is also true.

By induction, we conclude that \mathbf{X} , given by

$$\begin{aligned}
1) \quad x_{ij} &= v_{ij}^0 \text{ if } i < j \\
2) \quad x_{ij} &= v_{ji}^T \text{ if } i > j \\
3) \quad x_{ij} &= 1 - \sum_{i=1}^{j-1} v_{ij}^0 - \sum_{j=j+1}^n v_{ij}^T \text{ if } i = j
\end{aligned}$$

Which is non-zero and non-diagonal satisfies $\mathbf{X}\mathbf{e}' = \mathbf{e}' \forall i, j = 1 \dots n$

where

$\mathbf{e} = (f(q_1^T), f(q_2^T), \dots, f(q_n^T))$ with $f(q_j^T) = \partial q_j^T / \partial q_j^0$ and $v_{ij}^t = \frac{\partial q_j^t}{\partial q_i^0}$, $i, j = 1 \dots n$, $t = 0, T$ \square

⁶We now note some properties of the matrix-vector system which makes the usage fairly attractive for economists.

⁶Note that the theorem is consistent with Cramer's rule.

Notes

1. All the elements of our matrix are partial differentials valued with time being constant (i.e. each components of the matrix are specific to one and only one time point, either $t=0$ or $t=T$).
2. The elements of the vector are partial differentials relating to a single axis (i.e. each of the elements of the vector is specific to one and only one commodity i , i =either 1 or 2 or... or n). Furthermore, the growth vector, say $f(q_i^T) = \frac{\partial q_i^T}{\partial q_i^0}$ is not specified and the chain rule allows $f(q_1^T)$ to be, say, an exponential growth while $f(q_2^T)$ to be, say, a linear growth, and the general solution of the system of equations is given by:

$$\frac{v_{ij}^T}{v_{ij}^0} = f(q_j^T) / f(q_i^T) \quad (7.1)$$

7.2 A derivation with commodities

Suppose that we have an n -commodity economy where the measures of each commodity evolve through time t , each according to specific growth functions, $f(q_i^t)$, $i = 1 \dots n$. Without loss of generality, I suppose that the commodity bundle is $\kappa^t \in \Omega^n$, the n -dimensional Euclidean orthant and the physical quantity of commodity i at time t is denoted q_i^t . Given that we have a closed economy, the ratio of any arbitrary commodity i to another arbitrary commodity j , $i, j = 1 \dots n$ is fully specified at all times. Alternatively, given a system of ratios of all commodities to other commodities at different times, a unique vector of growths exists for each of the n commodities through time. My quest in this section is to specify a matrix whose entries are the ratios of commodity i to commodity j , $i, j = 1 \dots n$ at specific times, say $t = 0$ and $t = T$ that would correspond to the growth functions $f(q_i^T)$, $i = 1 \dots n$.

Suppose that we now, time $t = 0$, have q_i^0 of commodity i , $i = 1 \dots n$ which grows up to time T to q_i^T . I define the growth function, $f(q_i^T)$, $i = 1 \dots n$ to be the ratio of the future quantity of commodity i to its current quantity.

$$f(q_i^T) = \frac{q_i^T}{q_i^0} \quad (7.2)$$

Let Δq_{ij}^0 be the quantity of commodity i at time $t = 0$ that will be exchanged for (used in the production of) commodity j , at time $t = T$. Splitting commodity i into n parts at the present time, we have

$$q_i^0 = \sum_{j=1}^n \Delta q_{ij}^0 \quad (7.3)$$

Where Δq_{ij}^0 is the part of commodity i now that will make up for the part commodity j in the future. At that future time, T , the quantity of commodity j , q_j^T , $j = 1 \dots i \dots n$, is composed of the total parts from all n commodities, which at time $t = 0$ were allocated to its future production, Δq_{ij}^0 , $i = 1 \dots n$, each forwarded to time T at their respective growth rates, $f(q_i^T)$, $i = 1 \dots n$. The quantity of commodity j at time $t = T$ is then given by

$$q_j^T = \sum_{i=1}^n f(q_i^T) \Delta q_{ij}^0 \quad (7.4)$$

Now, from equations 7.2 and 7.3, we have

$$\begin{aligned} f(q_i^T) &= \frac{f(q_i^T) \sum_{j=1}^n \Delta q_{ij}^0}{\sum_{j=1}^n \Delta q_{ij}^0} \\ &= \frac{\sum_{j=1}^n f(q_i^T) \Delta q_{ij}^0}{\sum_{j=1}^n \Delta q_{ij}^0} \end{aligned} \quad (7.5)$$

And from equations 7.2 and 7.4, we have

$$\begin{aligned}
f(q_i^T) &= \frac{\sum_{j=1}^n f(q_j^T) \Delta q_{ji}^0}{q_i^0} \\
&= \frac{\sum_{j=1}^n f(q_j^T) \Delta q_{ji}^0}{\sum_{j=1}^n \Delta q_{ij}^0}
\end{aligned} \tag{7.6}$$

Equating equation 7.5 with 7.6,

$$\begin{aligned}
\sum_{j=1}^n f(q_i^T) \Delta q_{ij}^0 &= \sum_{j=1}^n f(q_j^T) \Delta q_{ji}^0 \\
\rightarrow \sum_{j=1}^n f(q_i^T) \Delta q_{ij}^0 - \sum_{j=1}^n f(q_j^T) \Delta q_{ji}^0 &= 0 \\
\rightarrow \sum_{j=1}^n (f(q_i^T) \Delta q_{ij}^0 - f(q_j^T) \Delta q_{ji}^0) &= 0
\end{aligned}$$

Let us consider the solution $f(q_i^T) \Delta q_{ij}^0 = f(q_j^T) \Delta q_{ji}^0 \forall i, j$. This solution is the equivalence relation that I shall investigate. In the next section, the continuous version of the equivalence relation, $\Delta q_{ij}^T = \Delta q_{ji}^T \forall i, j$, is shown to be a direct consequence of a major assumption in time preferences, namely the welfare-preserving rate. However, restricting ourselves to physical quantities for the time being, assuming that $f(q_i^T) \Delta q_{ij}^0 = f(q_j^T) \Delta q_{ji}^0 \forall j$, we have $\frac{f(q_1^T) \Delta q_{1i}^0}{\Delta q_{i1}^0} = \frac{f(q_2^T) \Delta q_{2i}^0}{\Delta q_{i2}^0} = \dots = \frac{f(q_n^T) \Delta q_{ni}^0}{\Delta q_{in}^0}$ implying that we can write $f(q_i^T)$, rather than a ratio of sums, as a sum of ratios⁷, i.e. equation 7.6 becomes $f(q_i^T) = \sum_{j=1}^n (\Delta q_{ji}^0 / \Delta q_{ij}^0) f(q_j^T)$. Thus, with $\frac{f(q_j^T) \Delta q_{ji}^0}{\Delta q_{ij}^0} = f(q_i^T) \forall j$, we have a system of linear equations which we can write in terms of a matrix of ratios, say $X = (x_{ij})$ where $x_{ij} = \frac{\Delta q_{ji}^0}{\Delta q_{ij}^0} i, j =$

⁷This follows from Euclid's proposition 12 (The elements, Book V). That is: "If any number of magnitudes be proportional, as one of the antecedents is to one of the consequents, so will all the antecedents be to all the consequents"

1..n and a vector of growths, say $\mathbf{e} = (e_1, e_2, \dots, e_n) = (f(q_1^T), f(q_2^T), \dots, f(q_n^T))$, such that $\mathbf{X}\mathbf{e}' = \mathbf{e}'$. I shall use the transpose notation to denote column vectors. Further, note that all elements of the Matrix X are ratios valued in the present time only, that is time $t = 0$.

Furthermore, given that we have a closed economy, with $\frac{f(q_j^T)\Delta q_{ji}^0}{\Delta q_{ij}^0} = f(q_i^T) \rightarrow \frac{f(q_i^T)\Delta q_{ij}^0}{\Delta q_{ji}^0} = f(q_j^T)$, our columns are also specified⁸. That is, given an entry in the upper triangular matrix, say x_{ij} , $j > i$, an entry in the lower triangular matrix, x_{ji} , is also specified such that $x_{ij}f(q_j^T) = x_{ji}f(q_i^T) \forall i, j$. Since it is more of our interest to consider how a single ratio changes, we wish to maintain the numerator and denominator so that they represent the ratio of the same commodities; but at different times. Thus if $x_{ij} = \frac{\Delta q_{ji}^0}{\Delta q_{ij}^0}$ $j > i$, and remembering that $x_{ij}f(q_j^T) = x_{ji}f(q_i^T) \forall i, j$, we let $x_{ji} = \frac{\Delta q_{ji}^T}{\Delta q_{ij}^T}$ so that it is only the time at which we consider the ratios that is changed⁹.

Next, with $f(q_j^T) = 1 \cdot f(q_j^T)$ implying that a unit of commodity j grows to $f(q_j^T)$, I add a further constraint on the diagonal entries. Given that we have $\mathbf{I}\mathbf{e}' = \mathbf{e}'$, then we firstly require that $\mathbf{X} = (x_{ij})$ $i, j = 1..n$ has an eigenvalue of one corresponding to the eigenvector, $\mathbf{e} = (f(q_1^T), f(q_2^T), \dots, f(q_n^T))$. We thus require that each of the columns of our matrix sum to 1, which is a con-

⁸Analogous to Sraffa's note relating to a system in a self-replacing state (page 5 of the production of commodity by means of commodity), our formulation presupposes a system undergoing indefinite growth. As a result such a state is feasible merely by changing the ratios in which the individual equations enter it.

⁹To maintain numerator and denominator of x_{ij} and x_{ji} , note that $\frac{\Delta q_{ji}^T}{\Delta q_{ij}^T}$ can be achieved by forwarding the numerator of the time zero ratio or by discounting the denominator of the time t ratio, i.e. $\frac{\Delta q_{ji}^T}{\Delta q_{ij}^T} = \frac{\Delta q_{ji}^0}{\Delta q_{ij}^0} f(q_j^T) = \frac{\Delta q_{ji}^T}{\Delta q_{ij}^T} f(q_i^T)$

sequence of equation 7.3. We therefore condition on the diagonal entries so that x_{jj} equal to $1 - \sum_{i, i \neq j} x_{ij}$. Thus we have specified all entries of our matrix of ratios such that $\mathbf{X}\mathbf{e}' = \mathbf{e}'$. That is,

$$\begin{pmatrix} 1 - \sum_{j=2}^n \frac{\Delta q_{j1}^T}{\Delta q_{1j}^T} & \frac{\Delta q_{21}^0}{\Delta q_{12}^0} & \cdots & \frac{\Delta q_{n1}^0}{\Delta q_{1n}^0} \\ \frac{\Delta q_{21}^T}{\Delta q_{12}^T} & 1 - \sum_{i=1}^1 \frac{\Delta q_{2i}^0}{\Delta q_{i2}^0} - \sum_{j=3}^n \frac{\Delta q_{j2}^T}{\Delta q_{2j}^T} & \cdots & \frac{\Delta q_{n2}^0}{\Delta q_{2n}^0} \\ \vdots & \vdots & \ddots & \vdots \\ \frac{\Delta q_{n1}^T}{\Delta q_{1n}^T} & \frac{\Delta q_{n2}^T}{\Delta q_{2n}^T} & \cdots & 1 - \sum_{i=1}^{n-1} \frac{\Delta q_{ni}^0}{\Delta q_{in}^0} \end{pmatrix} \mathbf{x} \begin{pmatrix} f(q_1^T) \\ f(q_2^T) \\ \vdots \\ f(q_n^T) \end{pmatrix} = \begin{pmatrix} f(q_1^T) \\ f(q_2^T) \\ \vdots \\ f(q_n^T) \end{pmatrix}$$

Thus far, however, we have not rigorously specified our matrix entries and vector entries. If we assume continuity in measures of the quantities of the commodity bundle at all times, without loss of generality, we could consider partial differentials rather than ratio of quantities¹⁰. Furthermore, such continuity assumptions often allow the incorporation of some economic definitions fairly well. For example, marginal substitution or marginal transformation can be well accommodated for. While the latter is not a requirement, it hardly seems unacceptable.

¹⁰For example $\frac{q_i^T}{q_i^T} q_i^T / q_i^0 \equiv \frac{\partial q_i^T}{\partial q_i^0} \partial q_i^T / \partial q_i^0$,

I shall let $f(q_i^t) = \partial q_i^t / \partial q_i^0$, $i = 1 \dots n$, $t = 0, T$ and consequently redefine the terms of our matrix, $\frac{\Delta q_{ji}^t}{\Delta q_{ij}^t}$, $i, j = 1 \dots n$, $t = 0, T$ in the traditional partial differential sense. Since the entries of X , only involve the pairwise exchanges, $\frac{\Delta q_{ji}^t}{\Delta q_{ij}^t}$, $i, j = 1 \dots n$, $t = 0, T$, i.e. the change in quantity of commodity i at time t that comes from commodity j at time 0 divided by the change in quantity of commodity j at time t that comes from commodity i at time 0, then given that $\partial q_j^t / \partial q_j^0 = f(q_j^t)$, $\frac{\Delta q_{ji}^t}{\Delta q_{ij}^t}$ can effectively be stated in a much simpler fashion; i.e. the change in commodity j with respect to commodity i at time t ¹¹, $\frac{\partial q_j^t}{\partial q_i^t}$. Next, suppose we wish to find the matrix inputs at a given time $t = T$ that corresponds to the vector e . Letting $v_{ij}^T = \frac{\partial q_j^T}{\partial q_i^T}$, the partial derivative of commodity j with respect to commodity i at the specific time T , we have the representation theorem.

7.3 An illustration in welfare

The non-specificity of an "absolute" function in our space opens doors for the use of the matrix system in various other sectors and for different other purposes. The mathematics of marginal rate of transformation and of marginal rate of substitution, being simply that of marginals or partial differentials, in this subsection, we wish to consider the term "marginal value" as input in our system. In order to justify the matrix approach for marginal substitutions, it is necessary to first make various simplifying assumptions, contingent on intergenerational equity. I shall follow Bergson's approach to the latter: namely through the selection of the welfare function, $W(\mathbf{q})$ (Bergson, 1938).

¹¹The use of partial differentials in the system provides a broader set of possibilities of definitions. However, restricting ourselves to the purpose of this chapter, we treat the usage as being a continuity of proportional change.

Although all social welfare functions have been much scorned for assuming interpersonal comparability of utility, intergenerational decision making seems unfeasible without making some assumptions about interpersonal comparability. Limiting cases of general social welfare functions are the Utilitarian social welfare which is a sum of utilities and the Rawlsian social welfare which is based on a *maxi-min* strategy¹² in which the welfare of the worst-off individuals ought to be maximized (Rawls, 1972). In this illustration, I shall assume the different discount rates are welfare-preserving.

The assumption of welfare-preserving rates is instructive because it offers a "spacial" expansion of the traditional framework for the social rate of time preference, the *srtp*. We recall that we have an economy that is closed and is composed of n commodities only; and we further require that our fictitious society satisfies the necessary assumptions required for the existence of an intercommodity and intertemporal indifference curve such as the Von Newman and Morgenstern (1953) rationality axioms and the axioms presented by Ok and Masatlioglu (2003), for intercommodity and intertemporal conditions respectively.

Definition 7.3.1. *Let the social welfare function, at time t , be $W^t \equiv W(q_1^t, q_2^t, \dots, q_n^t)$ where the commodity bundle is $\kappa^t \in \Omega^n$, the n -dimensional Euclidean orthant defined before.*

Definition 7.3.2. *Let the (negative) marginal substitution for commodity j between its future quantity and its current quantity be denoted as $f(Q_i^T) = \frac{W_i^t}{W_i^0} = -\frac{\partial q_i^0}{\partial q_i^t} \Big|_{dW=0}$, where $W_i^t = \partial W / \partial q_i^t$.*

Definition 7.3.3. *We further denote the (negative) social marginal rate of substitution between commodity j and commodity i at time τ , as $v_{ij}^\tau \equiv \frac{\partial W / \partial q_i^\tau}{\partial W / \partial q_j^\tau}$,*

Remarks. Definition 7.3.3, having a bijective nature, should remind us of the equivalence relation from the previous section; $f(q_i^T) \triangle q_{ij}^0 =$

¹²We discuss this in chapter 10

$f(q_j^T) \triangle q_{ji}^0 \quad \forall i, j$. However, in order to use the representation for preferences, it is not only required that we make assumptions of measurability for utility at all times but it is also necessary to assume the right conditions so that operations on quantities are valid for utility functions as well. Thus, to fully set the stage for the matrix approach, we shall require an independence condition introduced by Leontief (1947). To do so, we impose the equivalence lemma of Stigum (1967):

Definition 7.3.4. Let (K^τ, K^T) be a partition of the set of variables, K , and let \hat{K}^τ be any set of non-negative quantities of the variables in K^τ . The group of variables, K^τ , is separable in W from a variable K_k if and only if the correspondence, β , defined by $\beta(\hat{K}^\tau, K^T) = \{K^\tau \mid W(K^\tau, K^T) > W(\hat{K}^\tau, K^T)\}$, is independent of q_k , the quantity of the variable K_k .

Remarks. This definition is equivalent to the condition that the marginal rates of substitution at time τ , $v_{ij}^\tau \equiv \frac{\partial W / \partial q_i^\tau}{\partial W / \partial q_j^\tau}$, is independent of $q_k^t \forall t, i, j \neq k$ (Blackorby *et al*, 1973). If $W(K)$ is twice differentiable, then the condition is equivalent to $\frac{\partial v_{ij}^\tau}{\partial q_k^t} = 0$. Koopmans (1960) formulated the latter differently, though with similar implications. In some other classes of welfare models, time preferences are influenced by current consumption. As such, the marginal rate of substitution between commodities at time T and T' is dependent on consumption at time T'' and, consequently, well-being is not constant through time. However, our system considers only two time points and does not specify any absolute n -dimensional function. Consequently, assuming that the 2 sets of partial derivatives represent 2 time points, intertemporal marginal substitutions might be obtained simply from definition 7.3.2.

Since we again have two sets of partial differentials where each set, in turn, relate to each other through partial differentials, we can

write, with such definitions,

$$\sum_{j=1}^n x_{ij} f(q_j^T) = \sum_{j=1}^{i-1} x_{ij} f(q_j^T) + x_{ii} f(q_i^T) + \sum_{j=i+1}^n x_{ij} f(q_j^T)$$

where x_{ij} is defined as in theorem 7.1.1. and we have a consistent representation of marginal substitutions, which is based on the well-known framework for the *srtp*. We merely provide a matrix-vector terminology for ratios of a bundle of measures growing through time, whether the measure is physical quantity or a measure of preference that adhere to certain assumptions. With other welfare models, adjustments for $d\mathbf{w}$ are required ex-ante since our representation builds on the basis differential, $d\mathbf{q}$.

7.3.1 An application in health economic evaluations

The need for differential discounting in health economics was seen in the first part of this thesis to be, not only ethically correct, but also, to be empirically valid and theoretically motivated. Policies in the Netherlands and Belgium currently suggest differential discounting for health outcomes. We have also argued that inclusion of the pure rate, the rate at which future utility is discounted, is circular reasoning when discounting future lives is concerned, motivating a differential rate for health. While education, for example, is commonly known, with several empirical evidences, to boost both health and income, other economic activities are known, on the one hand, to be favourable to economic growth, while, on the other hand, to impact negatively on health; thus challenging the commonly assumed perfect correlation between health and income that justifies equal discounting of costs and effects.

This subsection focusses on the marginal values of health in terms of other commodities. Although we do not claim that health alone should be a measure of social welfare, we suggest that if, as Myrdal stated, production is a circular and cumulative sequence of causalities (Myrdal, 1963) then health is at least one of the explanatory

indicators of social welfare. This subsection shows that our theorem applies to current health discounting theory; i.e by assuming that our welfare function summarises a closed economy composed of health and income only.

Consistency with current health discounting

In order to validate our system with current health economic discounting, restricting ourselves, again, to welfare preserving rates for money and health, we shall use health economic literature notations (for this subsection only). Assuming that individuals make rational choices, we can assume that a three-dimensional indifference curve always exists. The health measures proposed have, most often, been a unique measure which is the combination of quality of life and life years, the QALY.

In case of an economy which is composed of only 2 commodities, the QALY and income only, our system provides the same result as current health economic approaches. In order to illustrate our matrix approach, we let commodity 1 and 2 be health, h , and money, m , respectively and use a one year time period, $(t = 0, t = T = 1)$, for this illustration. Let our social welfare function be summarised over only money streams and health streams, say $W(m^t, h^t)$ and the marginal value of future money in terms of current money be denoted by

$$\frac{W_m^1}{W_m^0} = - \frac{\partial m^0}{\partial m^1} \Big|_{dW=0} = \frac{1}{(1 + r_m)}$$

And let the marginal value of future health in terms of current health be denoted by

$$\frac{W_h^1}{W_h^0} = - \frac{\partial h^0}{\partial h^1} \Big|_{dW=0} = \frac{1}{(1 + r_h)}$$

Since we have only two commodities in such an economy, we need only consider one partial differential for the marginal substitution

among commodities, say the (negative) social marginal rate of substitution between money and health.

$$\frac{W_m^t}{W_h^t} = - \frac{\partial m^t}{\partial h^t} \bigg|_{dW=0} = v^t$$

where $W_h^t = \partial W / \partial h^t$ is the marginal social welfare from an infinitesimal increase in health at time t and $W_m^t = \partial W / \partial m^t$ is the marginal social welfare from an infinitesimal increase in money at time t .

From above theorem, in the case $n = 2$, we get the following system:

$$\begin{bmatrix} (1 - v^1) & v^0 \\ v^1 & (1 - v^0) \end{bmatrix} \begin{bmatrix} 1+r_h \\ 1+r_m \end{bmatrix} = \begin{bmatrix} 1+r_h \\ 1+r_m \end{bmatrix}$$

with solution,

$$\frac{v^0}{v^1} = \frac{1+r_h}{1+r_m}. \quad (7.7)$$

Note that we only expand one row of the matrix system to arrive at the solution due to linear independence, i.e.

$$(1 - v^1)(1+r_h) + v^0(1+r_m) = 1+r_h$$

$$\rightarrow -v^1(1+r_h) + v^0(1+r_m) = 0$$

$$\rightarrow \frac{v^0}{v^1} = \frac{1+r_h}{1+r_m}$$

That is, “*The marginal value of one good (health or income) in terms of another is the same whatever the route by which they are compared*”, as Gravelle and Smith (2001) stated. By considering the NPV of an intervention from two equivalent ways, Gravelle and Smith showed in a very straight forward way that v^0 and v^1 are related in the same linear fashion as above. They considered a single

one year period¹³ example where an intervention changes present and future costs by Δc^0 and Δc^1 respectively and the quantities of present and future health by Δh^0 and Δh^1 respectively. Firstly, they valued health effects in each period in terms of income and then discounted the future value at the rate of interest on income, r_m . The NPV of the intervention is given by

$$v^1 \Delta h^1 \frac{1}{(1+r_m)} + v^0 \Delta h^0 - \Delta c^1 \frac{1}{(1+r_m)} - \Delta c^0.$$

Secondly, they converted the change in future health into an equivalent change in current health and then applied the value of current health in terms of current income. This gives an NPV of

$$v^0 \Delta h^1 \frac{1}{(1+r_h)} + v^0 \Delta h^0 - \Delta c^1 \frac{1}{(1+r_m)} - \Delta c^0.$$

Then, by their consistency argument, equating the two NPV's yields

$$\frac{v^0}{v^1} = \frac{1+r_h}{1+r_m}$$

as we have found in equation 7.7. The reason why the matrix method is similar to the NPV method is that they are both solutions to the same problem. The aim was to find a relationship between v^0 and v^1 with the given constraints that $\partial h^1 = \partial h^0(1+r_h)$, $\partial m^1 = \partial m^0(1+r_m)$, and $v^t = \frac{W_m^t}{W_h^t}$. Thus in order to be indifferent to the gain of $(1+r_h)$ of future health at time $t = 1$, consistency requires that we could now, at time zero, hold either 1 unit of health only or $\left(\frac{v^0}{v^1}\right)$ units of income only or, in our case, we hold both income and health in the proportions: $(1 - v^1)$ units of health and v^0 units of wealth. We see that, in the example given by Gravelle and Smith, in order to have Δh^1 of health at time $t = 1$, they either hold $v^0 \Delta h^1 \frac{1}{(1+r_m)}$ of health

¹³Gravelle and Smith considered a period of time $t=0$ and a period of time $t=1$ and the definition they used was two-period while we use specific times $t=0$ and $t=1$ and hence assume a single period. The derivation however follows.

now or $v^1 \Delta h^1 \frac{1}{(1+r_h)}$ of income now. Thus, our 2x2 matrix and the NPV approaches provide the same results.

However, the matrix approach provides the opportunity for similar investigations in higher dimensions. Assuming interventions that improve health quality and interventions that increase life durations might also deserve differential discount rates from an ethical point of view, then we suggest that an *array-cost-effectiveness* analysis might be more theoretically defensible with the assumption of a commodity-specific discount rate.

7.4 Discussion

Although the assumptions of general equilibrium are known to allocate resources in a Pareto efficient manner, a major discussion is that a competitive market does not consider the growth of the economy in a sustainable manner. Pareto optimality is often said to fail to consider distributions among intertemporal societies in an ethical manner. It is thus unclear whether capital markets function in society's interest over periods spanning multiple generations. Moreover, discounting of human lives at the *SDR* has already been argued to lack the theoretical motivations. An alternative approach is proposed in this chapter by relating growth rates across commodities.

My approach resembles, to some degree, that of the original cyclical mechanisms that was proposed in Quesnay's *Tableau economique* (1759). I, however, rather than equating the "*physical quantity on the side of the means of production to that on the side of the product, both of which consist of the same product*" (Sraffa, 1960), allow for a non-fixed timing of the production process similar to Rae's *instruments*, which I equate through Euclid's proposition 12. It might not be unimportant to note that, while this chapter addresses consistency in preferences, using growths in physical quantities of a single product, investigations on a sustainable production-consumption cycle

seem fairly attractive. Alternatively, plugging in rates of time preferences as growth parameters of different commodities might aid in investigating consistency among social discount rates.

As Riccardo's methodology in devising rates of profits of a farmer by singling out corn as a 'basic' commodity, we choose to, rather, single out health measures as basic commodity. Analogous to Riccardo's conclusion with that regards, I propose that "*it is the growth in health that regulate the growth in other trades/commodities*". As a generic measure for health, the quality-adjusted life year (QALY) is often used. While the QALY is a multiplicative combination of health quality and life duration which is also consistent with health states that are worse than death or have zero duration of life, the assumption of linear utility of duration is often weakened for simplicity and challenges the actual discounting of the generic concept. We therefore suggest that the QALY be treated as its two different constituents, namely quality of life and the life year; which also strengthens the idea of an array-cost-effectiveness analysis. As such, our representation theorem opens the route to formally investigate potentially different discount rates for quality of life and life years which could especially be important for evaluating cost-effectiveness of life saving and life improving medical interventions differently.

7.a Appendix: Sraffa's production for subsistence approach to health

“Money makes money and the money that money makes makes more money”:-Benjamin Franklin (Hollis Page Harman, 1999). As a result of a society's production of income, we shall regard, in this Annex, income to be a commodity. The other two commodities that we shall consider to be contributing in the production of income are quality of life and life years so that a society with good health generates more income than one with poor health. Alternatively, a society with more income can make better investments towards life duration and quality of life. Furthermore, a society with no income and quality of life is assumed to perish, which is a reasonable assumption in today's world where the prime method of purchase is money. The three commodities, income, quality of life and life duration can thus be considered to be exchanged for one another to enable society to maintain itself.

To illustrate Sraffa's concept, we shall omit the value of time in this annex and consider the above mentioned 3-commodity economy to be closed. A short description of Sraffa's model for “production for subsistence” is as described below. Consider Quality of Life, Life years and Income as the commodities and let the annually produced Quality of Life in society be q_1 , that of Life years be q_2 and that of income be q_3 ; where part of the input of a commodity is transformed to part of the output of the other goods (including itself) and vice-versa. Furthermore, let q_{ij} be the annual quantity of good i used in producing q_j .

These quantities are assumed to be known by Sraffa. The unknowns to be determined are the values of the units of commodities, say p_1, p_2 and p_3 , which, if adopted, restore the initial position. We therefore have, since our economy is closed:

$$p_1q_{11} + p_2q_{21} + p_3q_{31} = p_1q_1$$

$$p_1q_{12} + p_2q_{22} + p_3q_{32} = p_2q_2$$

$$p_1q_{13} + p_2q_{23} + p_3q_{33} = p_3q_3$$

And, analogous to our columns summing to one due to the initial sum of all splits of a commodity should make the initial composition of that commodity, the same condition should be satisfied in Sraffa's matrix since his system is in a self-replacing state. Therefore, the conditions $\sum_{all\ j} q_{ij} = q_i \forall i$ are always satisfied. In Sraffa's words, "*The sum of the first column is equal to the first line, that of the second column to the second line, and so on.*" Then lastly, one commodity's value is taken as the standard value and it's price is made equal to one. It doesn't seem unreasonable to consider the value of income to be that of money. Thus, taking the price of income to be unity, we are left with three minus one unknowns which leaves 2 linearly independent equations uniquely determining p_1 and p_2 .

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Chapter 8

Probability Measures

The n -commodity economy presented, in the previous chapter, is however deterministic. To add probability measures, two methodologies were apparent with considerations on either the matrix entries or the vector entries. In order to attain face validation (validation due to conformity with current theoretical posits), I choose to consider the variability due the psychological limitations towards intertemporal choices. A key issue in modern neuroscience is the association that an individual's perception bears with her neural events (Crick and Koch, 2003). Patterns of intertemporal indifferences have captured the interest of psychologists and economists for several decades. Procrastination, addiction and willingness to save are a few observed behaviors involving time trade-offs (Dasgupta and Maskin, 2005). I shall thus assume that variability is centered around the weight functions in vector entries; where we shall **expect** an individual to be indifferent (\sim) between an amount, q^0 now and an amount $Q(T)$ at time $t = T$ to place a discount weight, $w(T) \in (0, 1)$ such that $q^0 \sim w(T)Q(T)$.

A very general discount weight function is of the form, $w(t) = e^{-\delta(t)\alpha(t)}$ where $\alpha(t)$ is the time-perception function and $\delta(t)$ is the preference rate function. Several authors have sought the proper functional form by eliciting either $\delta(t)$ or $\alpha(t)$ or both. Albrecht and

Weber (Albrecht and Weber, 1995), for example, proposed that individuals have a non-linear time preception function. The weight, $w(t)$, they proposed is given by $\delta(t) = \delta \forall t$ and $\alpha(t)$ is the non-linear time perception function indicating how fast time is perceived to pass in an individuals mind. Other authors assumed a linearly declining discount rate function, $\delta(t) = a + b(t)$ (Cropper *et al*, 1992) or a discontinuous hyperbolic discount weight function (Bazelon, 2002, Beutels *et al*, 2008) or an implicit function for the weights, $w(t)$ such as Loewenstein and Prelec's $w(t) = \frac{1}{(1+gt)^{\frac{h}{g}}}$ (Loewenstein and Prelec, 1992).

While there exist a wide body of literature suggesting the use of a single discount function, a series of studies suggest that cognitive considerations might explain the lack of convergence in empirical investigations. For example, our primate friend, *Saguinus oedipus*, and common marmosets, *Callithrix jacchus*, are known to differ in levels of temporal discounting (Rosati *et al*, 2006). Among the human species, we found in chapter 5 that health states, education level, to name a few, are endogeneous determinants of time preferences. Other studies have found that the speed of the internal clock is linked to the concentration of dopamine in the basal ganglia (Meck, 1996). However, similar to Samuelson (1937), I assume that the disparate psychological factors adding variability across an individual's intertemporal indifference can be condensed into the variability around her expected intertemporal indifference amount function. Then, I can analyse the sentence: "*In general, individuals prefer to consume now than to consume in the future*" with focus on the word "*in general*" and reformulate the latter sentence in a more statistically interpretable form. To do so, I condense all the variability into a mathematically tractable parameter; i.e. into the variability of *the specious present*.

Strictly speaking, the present is an infinitely small time interval separating the past and the future. However, it is argued that the per-

ceived present is an interval which need not be infinitely small, a concept first observed in 1882 (Clay in Holly and Grush, 2009). As Einstein described relativity with the faster passage of time when one sits in the company of a beautiful woman, it would also not be implausible that, everything else being constant, cognitive processes allow the "present time" to be viewed as a random variable. Furthermore, since the passage of time allows any future time interval to be constructed by a sequence of the individual's specious presents, then we have a time-inhomogeneous stochastic process similar to (Parouty *et al*, In Preparation). The assumption that an individual's perception of a future interval is a sequence of random specious presents coupled with the assumption that the individual is **expected** to place a weight on future consumption are sufficient to formulate a probabilistic distribution for intertemporal indifference.

8.1 Distribution derivation

The assignment of probability measures is done by deriving a maximum entropy distribution for the probability that an individual/animal is indifferent between some quantity $Q(T) = q$ at time $t = T$ and a given quantity $Q(0) = q^0$ now. First, I write our expectation in an elegant language, so that, given an initial amount, q^0 , now, the expected indifference amount at time t is

$$E(Q(t)) = \frac{q^0}{w(t)} \quad (8.1)$$

With equation 8.1 being the unconditional expectation of $Q(t)$, the Radon-Nykodym theorem asserts the existence of a conditional infinitesimal expectation (Varadhan, 2001). Defining $Q(t+h) - Q(t) =$

$\Delta_h Q(t)$, the latter is given by

$$\begin{aligned}
& \lim_{h \downarrow 0} \frac{1}{h} E(\Delta_h Q(t) \mid Q(t) = q) \\
&= \lim_{h \downarrow 0} \frac{1}{h} (E(Q(t+h) \mid Q(t) = q) - q) \\
&= \lim_{h \downarrow 0} \frac{q}{h} \left(1 + \left(\frac{-w'(t)}{w(t)} \right) h + o(h) - 1 \right) \\
&= \lim_{h \downarrow 0} \frac{q}{h} \left(\left(\frac{-w'(t)}{w(t)} \right) h + o(h) \right) \\
&= q \left(\frac{-w'(t)}{w(t)} \right)
\end{aligned} \tag{8.2}$$

Next, it is important to note that an individual or animal will only express a time preference provided she perceives time to pass (or provided the interval of time that she perceives is bigger than her specious present). Since the specious presents are random time durations, we can assume that a given fixed indifference amount at a specific time point has a random duration being equal to the individual's specious present at that time. Furthermore, at the instant that the next specious present begins, the indifference amount increases continuously with certainty. Thus, I ensure that an individual's intertemporal indifference path is a continuous function of time. As a result the process adheres to the typical characterisations of diffusion processes; usually restricted by a jump constraint on their probability measure, $P(Q(t) = q \mid Q(0) = q^0) \equiv P(q, t)$. To be more specific, for a positive ϵ sufficiently small,

$$\lim_{h \downarrow 0} \frac{1}{h} P(\Delta_h Q(t) > \epsilon \mid Q(t) = q) = 0. \tag{8.3}$$

Equation 8.3 ensures that the indifference path is time continuous. For example, an individual being given an amount q^0 now will always be indifferent to the same amount, q^0 , in any future interval which lies in her specious present. It is at the instant the new

specious present begins that the individual's indifference path exits the state q^0 moving, at all times, along a càdlàg path¹.

8.1.1 Probability assignment

I now wish to assign probability measures such that the *surprisal* or *hidden information* is maximized. Surprisal maximization is the standard principle in devising probability distributions. According to the principle of maximum-entropy, if we have a partial knowledge about a random variable(whether it is discrete or continuous, it's range, mean, etcetera), we first obtain a family of probability distributions that are all consistent with our information and then, we select, from that family, the single distribution whose uncertainty is the greatest. Most commonly used distributions are MaxEnt² given a current state of knowledge. For example, if we know nothing about a system of continuous random variables except it's range, we get the uniform distribution, or except it's positive mean, we get the exponential distribution, or except it's mean and standard deviation, we get the normal distribution, and so on.

Consequently, with the given infinitesimal mean, equation 8.2, I devise a MaxEnt distribution for a sufficiently small time interval, h , and thereafter projecting that distribution through time given the constraint that, for sufficiently small h , $E(\Delta_h Q(t) \mid Q(t) = q) \approx qh \left(\frac{-w'(t)}{w(t)} \right)$, to obtain a probability distribution. Note that the infinitesimal distribution, at any time, would be very much dependent on the weight function, $w(t)$, at that specific time. In the small enough time interval, h , in case of continuous intertemporal preferences, I consider the Dirac measure, $\mathbf{1}_{>0}(\Delta_h Q(t))$ which identifies³ an exit from a preference amount q in the interval $(t, t + h)$. The

¹Càdlàg means right continuous with left limits.

²MaxEnt is short for maximum-entropy.

³ $\mathbf{1}_{>0}(\Delta_h Q(t))$ takes a value of 1 if $\Delta_h Q(t) > 0$ and takes a value of 0 if $\Delta_h Q(t) = 0$

Lagrangian is given by

$$\begin{aligned} & \mathcal{L}(P(\mathbf{1}_{>0}(\Delta_h Q(t)) \mid Q(t) = q), \lambda, \beta) \\ &= \int P(\mathbf{1}_{>0}(\Delta_h Q(t)) \mid Q(t) = q) \ln(P(\mathbf{1}_{>0}(\Delta_h Q(t)) \mid Q(t) = q)) dq \\ & - \beta \left(\int \mathbf{1}_{>0}(\Delta_h Q(t)) P(\mathbf{1}_{>0}(\Delta_h Q(t)) \mid Q(t) = q) dq - qh \left(\frac{-w'(t)}{w(t)} \right) \right) \\ & - \lambda \left(\int P(\mathbf{1}_{>0}(\Delta_h Q(t)) \mid Q(t) = q) dq - 1 \right) \end{aligned}$$

Differentiating $\mathcal{L}(\cdot)$ with respect to $P(\cdot)$ yields $P(\mathbf{1}_{>0}(\Delta_h Q(t)) \mid Q(t) = q) = e^{-\lambda - \beta(\mathbf{1}_{>0}(\Delta_h Q(t)))^{-1}}$. Picking λ and β so that $P(\cdot)$ is a probability measure and $E(\Delta_h Q(t) \mid Q(t) = q) = qh \left(\frac{-w'(t)}{w(t)} \right)$ gives

$$P(\Delta_h Q(t) = 0 \mid Q(t) = q) = 1 - qh \left(\frac{-w'(t)}{w(t)} \right) \quad (8.4)$$

$$P(\Delta_h Q(t) > 0 \mid Q(t) = q) = qh \left(\frac{-w'(t)}{w(t)} \right) \quad (8.5)$$

It is interesting to remark that the use of the Dirac measure above assumes unit jumps and hence equations 8.4 and 8.5 provide a good analogy for countable indifference amounts, such as euros⁴. In the appendix of this chapter, I derive a discrete state, continuous time, process, namely the Poisson process. Equation 8.1 however requires that $Q(t)$ be continuous and thus hints towards the use of differentials in probability with respect to q . I do so, however, after formulating the Chapman-Kolmogorov equations.

⁴That is, for example, given an individual's indifference amount, $Q(t) = q$ at time t , the probability that in the small time interval, $(t, t+h)$, the individual will not demand any more money is $P(\Delta_h Q(t) = 0 \mid Q(t) = q) = 1 - qh \left(\frac{-w'(t)}{w(t)} \right)$ and the probability that the individual will demand €1 in the interval, h , is $P(\Delta_h Q(t) = 1 \mid Q(t) = q) = qh \left(\frac{-w'(t)}{w(t)} \right)$ which is a valid Bernoulli distribution.

8.1.2 Differential equation

The general Chapman-Kolmogorov equation is given by

$$P(q, t + h) = \int_{allz} P(q - z, t) P\{\Delta_h Q(t) = z \mid Q(t) = q - z\} dz \quad (8.6)$$

In our case, since we have only two possible outcomes in the interval h , no growth or an infinitesimal growth as specified in equation 8.2. Our Chapman-Kolmogorov is then approximately:

$$\begin{aligned} P(q, t + h) &= P(\Delta_h Q(t) = \epsilon \mid Q(t) = q - \epsilon) P(q - \epsilon, t) \\ &\quad + P(\Delta_h Q(t) = 0 \mid Q(t) = q) P(q, t) \\ &= (q - \epsilon) h \left(\frac{-w'(t)}{w(t)} \right) P(q - \epsilon, t) \\ &\quad + \left(1 - qh \left(\frac{-w'(t)}{w(t)} \right) \right) P(q, t) \end{aligned}$$

giving

$$\frac{P(q, t + h) - P(q, t)}{hP(q, t)} = q \left(\frac{-w'(t)}{w(t)} \right) \left(\frac{q - \epsilon}{q} \frac{P(q - \epsilon, t)}{P(q, t)} - 1 \right)$$

Consider the backward difference operator, $\nabla_\epsilon u(q) = u(q) - u(q - \epsilon)$. Letting $g(q) = \ln(q)$ and $L(q, t) = \ln(P(q, t))$, the Chapman-Kolmogorov equation becomes,

$$\begin{aligned} \frac{P(q, t + h) - P(q, t)}{hP(q, t)} &= q \left(\frac{-w'(t)}{w(t)} \right) (e^{-\nabla_\epsilon g(q) - \nabla_\epsilon L(q, t)} - 1) \\ &= q \left(\frac{-w'(t)}{w(t)} \right) \left(e^{\sum_{n=1}^{\infty} \frac{(-\epsilon)^n}{n!} \frac{\partial^n g(q)}{\partial q^n} + \sum_{n=1}^{\infty} \frac{(-\epsilon)^n}{n!} \frac{\partial^n L(q, t)}{\partial q^n}} - 1 \right) \end{aligned}$$

Now, since $g(q)$ and $L(q, t)$ are both assumed to be analytic and successively differentiable upto some order, using Cauchy's mean value theorem, we can write

$$\sum_{n=1}^{\infty} \frac{(-\epsilon)^n}{n!} \frac{\partial^n g(q)}{\partial q^n} + \sum_{n=1}^{\infty} \frac{(-\epsilon)^n}{n!} \frac{\partial^n L(q, t)}{\partial q^n} = -\frac{\partial L(q, t)}{\partial q} \epsilon - \frac{\partial f(q)}{\partial q} \epsilon$$

where $f(q)$ is some analytic function⁵. The Chapman-Kolmogorov equation can thus be written as

$$\frac{P(q, t+h) - P(q, t)}{hP(q, t)} = q \left(\frac{-w'(t)}{w(t)} \right) \left(e^{-\frac{\partial L(q, t)}{\partial q} \epsilon - \frac{\partial f(q)}{\partial q} \epsilon} - 1 \right) \quad (8.7)$$

Now, taking $\lim_{h \downarrow 0}$ of equation 8.7 and recalling that we used the Dirac measure gives:

$$\frac{\partial L(q, t)}{\partial t} = q \left(\frac{-w'(t)}{w(t)} \right) \left(e^{-\frac{\partial L(q, t)}{\partial q} - \frac{\partial f(q)}{\partial q}} - 1 \right) \quad (8.8)$$

8.1.3 Distribution

Equation 8.8 can be readily solved through standard methods (Forsyth, 1929). We note that the equations of the tangent planes to the surface satisfying equation 8.8 at arbitrary points l_0, q^0, t_0 form an envelope which is the actual surface $L(q, t)$. Then taking $\frac{\partial L(q, t)}{\partial t} = \varphi(\alpha, t)$, $\frac{\partial L(q, t)}{\partial q} = \psi(q, \alpha)$ and treating α as a constant, we no longer have a partial differential equation. We can thus let $l_\varphi(\alpha, t)$ and $l_\psi(q, \alpha)$ be the solutions to each, respectively, and obtain the complete integral which is a two parameter family of planes:

$$L(q, t) = l_\varphi(\alpha, t) + l_\psi(q, \alpha) + c \quad (8.9)$$

where c is a constant independent of q and t . Writing $\varphi(\alpha, t) = \alpha \left(\frac{-w'(t)}{w(t)} \right)$, we get

$$\begin{aligned} l_\varphi(\alpha, t) &= \int_0^t \alpha \left(\frac{-w'(t)}{w(t)} \right) dt \\ &= \alpha \ln \frac{1}{w(t)} \end{aligned} \quad (8.10)$$

⁵Note that the assumption of a general analytic function provides more room towards later formulating a normalizing constant.

Next, with $\varphi(\alpha, t)$ given as above, we have $\alpha = q \left(e^{-\frac{\partial L(q,t)}{\partial q} - \frac{\partial f(q)}{\partial q}} - 1 \right)$

which implies $\psi(\alpha, q) = \ln \left(\frac{qe^{-\frac{\partial f(q)}{\partial q}}}{q+\alpha} \right)$ with solution

$$\begin{aligned} l_\psi(q, \alpha) &= \int_{q^0}^q \ln \left(\frac{q}{q+\alpha} \right) - \frac{\partial f(q)}{\partial q} dq \\ &= \int_{q^0}^q \ln(q) - \ln(\alpha + q) - \frac{\partial f(q)}{\partial q} dq \\ &= q \ln(q) - q^0 \ln(q^0) - (q + \alpha) \ln(q + \alpha) \\ &\quad + (q^0 + \alpha) \ln(q^0 + \alpha) - f(q, q^0) \end{aligned} \quad (8.11)$$

Substituting equations 8.11 and 8.10 into equation 8.9 gives

$$\begin{aligned} L(q, t) &= q \ln(q) - q^0 \ln(q^0) - (q + \alpha) \ln(q + \alpha) \\ &\quad + (q^0 + \alpha) \ln(q^0 + \alpha) - f(q, q^0) + \alpha \ln \frac{1}{w(t)} + c \end{aligned} \quad (8.12)$$

Next, we need to eliminate α from equation 8.12 which we do by giving α a pair of equal values for the same values of $l_\psi(q, \alpha)$ and $l_\varphi(\alpha, t)$. We thus form the equation $\frac{\partial L(q,t)}{\partial \alpha} = 0$, giving

$$\ln \frac{1}{w(t)} + \ln \left(\frac{\alpha + q^0}{\alpha + q} \right) = 0 \quad (8.13)$$

from which, we get

$$\alpha = \frac{q - \frac{q^0}{w(t)}}{\frac{1}{w(t)} - 1} \quad (8.14)$$

Again, substitution of α from equation 8.14 into equation 8.12, gives, after some algebraic simplifications,

$$L(q, t) = (c - f(q, q^0)) + \ln \left(\frac{q^q}{(q^0)^{q^0} (q - q^0)^{q - q^0}} \right) w(t)^{q^0} (1 - w(t))^{q - q^0} \quad (8.15)$$

resulting in

$$P(q, t) = \frac{e^{c-f(q, q^0)} q^q}{(q^0)^{q^0} (q - q^0)^{q - q^0}} w(t)^{q^0} (1 - w(t))^{q - q^0} \quad (8.16)$$

Integrability on the probability space, (E, ξ) , requires picking c and $f(q, q^0)$ so that $P(\cdot)$ is a probability measure. Letting $f(q, q^0)$ be the truncated parts of Binet's formula for reals (Zoltan, 1999),

$$f(q, q^0) = \int_0^\infty \left(\frac{1}{e^x - 1} - \frac{1}{x} + \frac{1}{2} \right) \frac{e^{-qx} - e^{-(q - q^0)x}}{x} dx + \frac{1}{2} \ln \left(\frac{q - q^0}{q} \right) - q^0$$

and putting $c = \ln \frac{(q^0)^{q^0}}{\Gamma(q^0 + 1)}$, we have a Gamma-Poisson mixture type distribution

$$P(q, t) = \frac{\Gamma(q)}{\Gamma(q^0 + 1) \Gamma(q - q^0)} w(t)^{q^0} (1 - w(t))^{q - q^0} \quad (8.17)$$

8.1.4 Remarks

It's not unimportant to note that the probability distribution is not a distribution for **time preferences** but rather a distribution for **intertemporal indifferences**. To be more precise, I find a closed-form distribution for the probability that an individual will actually be indifferent between some quantity of $Q(t) = q$ at time t and a given quantity $Q(0) = q^0$ now and the individual does not *imagine*, today, a future quantity that she feels she will be indifferent to, compared to a given current quantity $Q(0) = q^0$. The distribution thus cannot be applied in chapter 5; although knowledge of the exact variability in the neuro-networks involved in the construction of time in an individual's mind could allow for a proper mathematical writing. Although I have assumed, thus far, an individual prespective where the individual lives forever, societal perspectives are, however, common recommendations.

It is thus necessary that I discuss the assumptions under which my distribution is applicable to society. Two assumptions appear sufficient: first, I ought to assume that society lives forever⁶ and second, I ought to assume that society possesses a societal *sense of time* as with the "average" individuals comprising it. Individuals with Parkinson's disease, attention deficit hyperactivity disorder (ADHD) and schizophrenia have a marked difference in time perception compared to an "average" individual (Davalos *et al*, 2002, Levy and Swanson, 2001, Pastor *et al*, 1992). The sense of time is often argued to be a consequence of the dopaminergic state, influencing the sensory signal-to-noise ratio due to an effect of dopamine at the cellular level (Williams and Goldman-Rakic, 1995, Winterer *et al*, 2000).

Consequently, even the "average" individual is bound to experience a marked alteration in time perception if drugs, such as psilocybin (Wittman *et al*, 2007), increasing the function of dopamine⁷, become common usage. Although the latter appears farfetched, an economic scenario entailing a general alteration in the "average" individual's time perception appears reasonably realistic. Neurobiologists and psychologists often make the following associations: opioid = pleasure, dopamine = happiness, serotonin deficit = depression, oxytocin = love, nucleus accumbens = reward or amygdala = fear, etcetera (Kringelbach and Berridge, 2003). Economists, on the other hand, would coin a *bullish* or a *bearish* sentiment, reflecting a positive or a negative feeling about market trends. A society with a bull market, a market with an upward trend, for example, might experience time to pass faster than one with a market with a downward trend, a bear market, say⁸.

⁶I will assume that, through our children, we renew the link that binds the chain of our humanity through time and attain "some kind of infinity". A specific type of infinity, as an example, here, is an *actual infinity* if we can assume that time is an entity.

⁷While dopamine is known to play a role in both our reward system and our time perception, norepinephrine and serotonin are also argued affect the sense of time.

⁸Bull and bear are the symbolic beasts of finance.

As such, I assume some *ethical*⁹ weights among the preferences of individuals comprising a temporal society so that we can assume that society, as a whole, also experiences the passage of time as a sequence of societal specious presents. Then a society will be indifferent to a single fixed quantity for the duration of its specious present, before feeling the need for some extra amounts, with a social rate of time preference, *srt**p*, when it perceives the present to have become past. Moreover, we can also assume non-subjective rates.

Although, my considerations, thus far, have only concerned a subjective rate of time preference, in the case where rates are not subjective, such as the rate on government bonds, without loss of generality, the probability of intertemporal indifference for a society with an *external* social discount rate, *SDR*, can be equivalently seen to follow the same distributional laws as given in equation 8.8 since the main source of variability is assumed to be in the perception of time. To be specific, the expectation is taken as given and it is only the occurrence times of events, identified by zeroes or ones, that are random and assigned probability measures through surprisal maximization. So, society's probability of intertemporal indifference, $P(Q(t) = q/Q(0) = q^0) \equiv P(q^t, q^0)$ can be represented by equation 8.17.

8.2 Probability of the future monetary value of health

The distribution derived in the previous section is noted to construct on the Dirac measure, $1_{>0}(\Delta_h Q(t))$, which identifies an exit from

⁹The term *ethical* is used here because of the numerous discussions regarding whose preferences should count. As we found in chapter 5, educated individuals generally have different rates of time preference compared to uneducated ones. Such discussions are, however, beyond the scope of this thesis.

a preference amount q in the interval $(t, t + h)$. Consequently, the infinitesimal distribution, given by equations 8.4 and 8.5, also satisfy discrete increments¹⁰. That is

$$P(\Delta_h Q(t) = 0 \mid Q(t) = q) = 1 - qh \left(\frac{-w'(t)}{w(t)} \right) \quad (8.18)$$

$$P(\Delta_h Q(t) = 1 \mid Q(t) = q) = qh \left(\frac{-w'(t)}{w(t)} \right) \quad (8.19)$$

are probability measures that maximise the entropy in the time interval, $(t, t + h)$ given the constraints aforementioned. Thus, although $P(\cdot)$, given by equation 8.17, is a continuous state process, we considered increments of one and allowed a continuous function to pass through those increments by the use of differentials. As a result, the discrete version of equation 8.17, is obtained by converting the $\Gamma(\cdot)$ functions to their factorial equivalents, obtaining a Yule-Furry type process:

$$P(Q(t) = q) = \binom{q-1}{q^0} w(t)^{q^0} (1 - w(t))^{q-q^0} \quad (8.20)$$

Thus, for an individual considering outcomes that are discrete such as the Euro or the QALY, equation 8.20 gives the probability that the individual is indifferent between some quantity $Q(t) = q$ at time $t = T$ and a given quantity $Q(0) = q^0$ now given a discount weight, $w(t)$.

With a distribution for the probability of intertemporal indifference, it is feasible to make probabilistic statements about the monetary value of health in the future. We recall that differential discounting is pedantically establishable provided the monetary value of health is non-constant with respect to time. Let the probability that an individual is indifferent between some quantity of money $M(t) = m$ at a future time t and a given quantity of money $M(0) = m^0$ now be

$$P(M(t) = m) = \binom{m-1}{m^0} w(t)^{m^0} (1 - w(t))^{m-m^0}$$

¹⁰See appendix of this chapter for a discrete state continuous time process.

and let the probability that the individual is indifferent between some measurable quantity of health $H(t) = h$ at a future time t and a given quantity of health $H(0) = h^0$ now be

$$P(H(t) = h) = \binom{h - 1}{h^0} v(t)^{h^0} (1 - v(t))^{h - h^0}$$

where the function $v(t)$ is possibly different from $w(t)$. Given the current monetary value of health, $k = \frac{m^0}{h^0}$, we wish to find the probability that at a future time, t , the monetary value of health will be the same. With $M(t)$ and $H(t)$, being random variables, the monetary value of health is also a random variable. Let us consider the change of variables

$$Y(t) = \frac{M(t)}{H(t)} \quad (8.21)$$

and

$$X(t) = H(t) \quad (8.22)$$

We wish to find the distribution of $Y(t)$. Since $M(t)$ and $H(t)$ are two independent random variables, their joint p.d.f is given by $P(M(t) = m, H(t) = h) = P(M(t) = m) \times P(H(t) = h)$. Now, with $M(t) = Y(t)X(t)$ and $H(t) = X(t)$, and the Jacobian

$$\mathbf{J} = \begin{vmatrix} x & y \\ 0 & 1 \end{vmatrix} = x$$

the joint distribution of $Y(t)$ and $X(t)$ is given by

$$P(Y(t) = y, X(t) = x) = |J| P(Y(t)X(t) = yx, X(t) = x)$$

which equals

$$\begin{aligned} P(y, x) &= \binom{xy - 1}{xy^0} w(t)^{xy^0} (1 - w(t))^{xy - xy^0} \\ &\quad \times \\ &\quad \binom{x - 1}{x^0} v(t)^{x^0} (1 - v(t))^{x - x^0} \\ &\quad \times \\ &\quad x \end{aligned}$$

The distribution of $Y(t)$ can thus be obtained by marginalizing out $X(t)$; i.e. $P(Y(t) = y) = \sum_{x=x_0}^{\infty} P(Y(t) = y, X(t) = x)$. The computation, however, requiring complex mathematics¹¹, might be numerically approximated using today's advanced softwares. Alternatively, rather than naively finding the solution, one might attempt to bound the probability, depending on one's objectives.

8.3 Discussion

I previously mentioned my aim to achieve face validation with current psychological theory. Let me discuss the construction of the probability distribution, in section 8.1, which stands mainly on the grounds of the philosophy of mathematics. In Particular, the principle that I have observed, in contrast to Laplace's Principle of Insufficient Reason mentioned in chapter 3, is known as Leibniz's Principle of Sufficient Reason. Leibniz's principle, often associated with *ex nihilo nihil fit*¹², has been influential in the thinking of several philosophers and was recognised as one of the four laws of thought in the 18th century. The principle states that the occurrence/existence/truthfulness of an event/entity/statement imply the occurrence/existence/truthfulness of a sufficient explanation.

In this chapter, I have repeatedly made use of this principle. Among the different senses of the human being, time perception is a sense that is increasingly gaining the interest of psychologists. Given that perceived passage of time is not equal to hour-glass passage of time and that individuals prefer to consume now than in the future imply the existence of an intertemporal indifference probability distribution. Furthermore, let us assume that we observe the individual for an instant, or, say, for a sufficiently small time interval¹³. At that

¹¹The main problem is the quasi-absolute lack of literature on $\sum_{x=x_0}^{\infty} \binom{xy}{x^0 y^0}$

¹²A latin expression, argued in the first thesis of Parmenides, 5th century BCE, meaning *nothing comes from nothing*.

¹³One could take the standard unit of time, the second, defined to be the duration

instant, either the individual would have perceived time to pass or not and, consequently, either there exists an exit from a preference amount or not; wherefore I obtained equations 8.4 and 8.5 which formed the infinitesimal distribution. Projecting that distribution through time, I deduced equation 8.17. Summarising, the derivation of 8.17 is based on two assumptions; namely:

1. An individual is expected to value a future quantity by placing a discount weight on that quantity, i.e. $E(Q(t)) = \frac{q^0}{w(t)}$
2. An individual experiences the passage of time as a sequence of specious presents.

A point, perhaps, needs mentioning. Another important assumption that I made is that I have assumed that the individual/society lives forever implying that a choice can always be made. In case of investigations involving mortality, the latter can be rectified by taking into account mortality tables or parametric mortality laws and conditioning the probability of a choice on the probability that the choice exists. That specification is beyond this thesis since it is more useful for us to assume that the individual lives forever.

of 9,192,631,770 periods of the radiation corresponding to the transition between the two hyperfine levels of the ground state of the caesium 133 atom, as an approximation.

8.a Appendix: The Poisson process

Since I used the Dirac measure, let me derive, in similar vein, a discrete state continuous time process. A particularly important integer-valued process is the Poisson process¹⁴. Along with providing analogies to my derivation, this appendix shall derive a Poisson process with intensity λ ; i.e. a Poisson distribution with mean λt . The intensity (or mean rate or infinitesimal mean) is analogous to equation 8.2

$$\lim_{h \downarrow 0} \frac{1}{h} E(\Delta_h Q(t) \mid Q(t) = q) = \lambda \quad (8.23)$$

Note that the parameter does not depend on q or t , which simplifies much of the remaining computations¹⁵. We shall assume that the possible transitions from a certain state in the small enough time interval, h , are either no increments at all or a unit increment. So, from a similar Lagrangian, we find the analogies to equations 8.4 and 8.5 as

$$P(\Delta_h Q(t) = 0 \mid Q(t) = q) = 1 - \lambda h + o(h) \quad (8.24)$$

$$P(\Delta_h Q(t) = 1 \mid Q(t) = q) = \lambda h + o(h) \quad (8.25)$$

I should also mention that increments that are larger than unity are here assigned the probability $o(h)$, the Landau order¹⁶, which can be ignored. The Chapman-Kolmogorov equation 8.6 can be written, in this case, by noting that an outcome q at time $t+h$ can be realized by, either having an outcome q at time t and no increments in the interval h or having an outcome $q - 1$ at time t and a unit increment in the

¹⁴Karlin and Taylor (1975) provide a thorough derivation.

¹⁵Such a non-dependence often coins a homogeneous process in contrast to our inhomogeneous process.

¹⁶The Landau order, $o(t) = f(t), t \rightarrow 0$ is the symbolic way of writing the relation $\lim_{t \rightarrow 0} f(t)/t = 0$. I shall, however, not discuss the mathematical details here.

interval h or having an outcome $q - 2$ at time t and two increments in the interval h and so on. So, we can write 8.6 as

$$\begin{aligned} P(q, t + h) &= \sum_{i=0}^q P(q - i, t) P(\Delta_h Q(t) = i \mid Q(t) = q - i) \\ &= P(q - 1, t)(\lambda h + o(h)) + P(q, t)(1 - \lambda h + o(h)) + o(h) \end{aligned}$$

From the properties of $o(h)$, we can write

$$P(q, t + h) = P(q - 1, t)\lambda h + P(q, t) - P(q, t)\lambda h + o(h) \quad (8.26)$$

With a change of subject of formula, we obtain the analogy for 8.7 as

$$\frac{P(q, t + h) - P(q, t)}{h} = P(q - 1, t)\lambda + -P(q, t)\lambda + \frac{o(h)}{h} \quad (8.27)$$

Taking the limit $h \downarrow 0$, akin to 8.8,

$$\frac{\partial P(q, t)}{\partial t} = P(q - 1, t)\lambda + -P(q, t)\lambda \quad (8.28)$$

Since the Poisson process is a counting process, I shall assume that one starts counting at time $t = 0$. So we have an initial condition that $P(q, 0) = 0$ for $q > 0$. Solving 8.28, in contrast to devoting the whole of subsection 8.1.3 towards solving 8.8, uses a simple trick. We introduce the function, $R(q, t) = P(q, t)e^{\lambda t}$, which we substitute in 8.28 to give

$$\frac{\partial R(q, t)}{\partial t} = \lambda R(q - 1, t) \quad (8.29)$$

with $R(0, t) \equiv 1$, $R(q, 0) = 0$ and $q = 1, 2, \dots$. Solving 8.29 recursively, we obtain

$$\begin{aligned}\frac{\partial R(1, t)}{\partial t} &= \lambda R(0, t) \longrightarrow R(1, t) = \lambda t \\ &\longrightarrow R(2, t) = \frac{(\lambda t)^2}{2} \\ &\longrightarrow R(3, t) = \frac{(\lambda t)^3}{3!} \\ &\quad \vdots \\ &\longrightarrow R(q, t) = \frac{(\lambda t)^q}{q!}\end{aligned}$$

Therefore,

$$P(q, t) = P(Q(t) = q) = \frac{(\lambda t)^q}{q!} e^{-\lambda t} \quad (8.30)$$

So, for each t , $Q(t)$ follows a Poisson distribution with parameter λt . Our process, given by 8.17, is derived in similar fashion to the Poisson process since the specious present is a random duration. However, further elaboration of the Poisson example is not relevant as distributions suitable for discounting would uniquely follow the specifications in section 8.1, with different weight functions. Yet, the derivation of the Poisson process illustrates some of the key equations that are obtained. The occurrence times of an event, being the sole source of variability that were assumed, allows for various other applications such as population growth, arrival rates, etcetera.

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Chapter 9

An Aspiration Avenue

Having a deterministic system of marginals in chapter 7 and a stochastic distribution for each of the system's entries in the previous chapter, this chapter marries the two and discusses future aspirations. The future is uncertain, at least, on the basis of the senses we have been endowed with. The problem of devising decision rules under uncertainty, probably, dates back to the dawn of humanity. I, however, aspire to begin this chapter in the early 1700, when decisions under the conditions of risk and uncertainty gained much attention, through the representation of lotteries. In the early days of probability theory, the value of a lottery with a random outcome, Q , was accepted to be it's mathematical expectation:

$$\int_{all\ q} qf(q) dq$$

where $f(q)$ is the associated probability density function¹. Consequently, a fair value of the lottery was it's expected value. In 1738, Daniel Bernoulli was presented with a case when the integral above

¹Accordingly, $F(q)$ will denote the cumulative probability function.

does not converge² wherefore the lottery has no fair value³. The latter would refrain a rational individual from partaking in this lottery, and yet, betters were willing to enter the game at arbitrary amounts⁴. This was coined as the St. Petersburg Paradox. To solve this, Bernoulli (1738) suggested that the value of a particular payoff was not directly related to its precise monetary worth but to a subjective value. He proposed a monotonically increasing function of q that allowed the integral to converge. The fair value of the lottery was then suggested to be its *moral expectation*:

$$\int_{all\ q} \ln(q) f(q) dq$$

The suggestion of Bernoulli (1738) was that an individual assigns a *moral value* of $\ln(q)$ to a reward of q . Although prior to the beginning of the 18th century, decisions under uncertainty primarily sought the expected value of an outcome, involving solely a probability function, when Bernoulli (1738) considered valuations based on the concept of the outcome's utility, the latter hatched interesting ideas about using, more general, "moral" functions.

About two centuries later, Von Neumann and Morgenstern (1944) went about the problem from a different direction. Through a few axiomatic assumptions about an "economic" agent with a well-organised and stable system of preferences, they proposed the Expected Utility Theory(subjective expected utility). The axioms⁵, presented in

²The case was presented by Daniel's cousin, Nicolas Bernoulli; both, nephews of Jacob Bernoulli who is known for the Bernoulli distribution mentioned in the previous chapter.

³Briefly, the game entails a reward of 2^q if the first head appears on the q^{th} tossing of a fair coin. The expected reward is $E(q) = \frac{1}{2}2 + \frac{1}{4}4 + \frac{1}{8}8 + \dots = \infty$, and hence this game has no fair value.

⁴In contrast to this game, where the expectation at each toss is given, the distribution I propose, rather, takes an expectation dependent on time, as given, and formulates Bernoulli probability measures for each instantaneous Bernoulli distributions.

⁵Axioms are not necessarily related to time; so T, T' and T'' can be purely viewed

section 4.2, ultimately gained more considerations over the last few decades through Ellsberg (1961), Karmakar (1978), Kahneman and Tversky (1982), Fishburn (1983), to name a few. With that regards, this chapter seeks primarily to discuss the distribution derived in the previous chapter, assuming a more general unit of measurement, the util.

9.1 Utility and aspiration

Utility is an interesting word. Even the founder of modern Utilitarianism, Jeremy Bentham (1789) remarked that his previous works would have been better understood had he used the words happiness or felicity. A utility function serves to map an "external" good, through our senses, to an "internal" real-valued function of happiness, calibrated by the util. Whether our happiness can be mapped onto a real-valued function is, however, much debated. Elster and Roemer (1991) provide a bewildering array of such issues, especially with regards to interpersonal comparability of utility. The isoelastic utility function, $U(q) = \frac{q^{1-\varepsilon}}{1-\varepsilon}$, mentioned in chapter 3, Bernoulli utility function mentioned above, $U(q) = \ln(q)$, exponential utility function, $U(q) = 1 - e^{-ac}$, are a few of those functions that assign a real value to our consumption, q .

Given a lottery with random variable, Q , a utility function, $U(q)$, and a probability distribution, $f(q)$, the expected utility of the lottery is given by

$$\int_{all\ q} U(q)f(q)dq \quad (9.1)$$

The certainty equivalent, \hat{q} , of this lottery is the value of q that provides the same utility as equation 9.1; i.e. $U(\hat{q}) = \int_{all\ q} U(q)f(q)dq$. An alternative way in deriving the certainty equivalent replaces the cumulative function, $F(q)$, with a step cumulative probability

as indices.

yielding the same expected utility. A dual notion is the aspiration equivalent, \tilde{q} , where, in this case, the utility function is replaced by a step utility function yielding the same expected utility.

9.1.1 The Probabulity concept

The dual notion arises primarily due to the dual nature of the very problem: maximization of expected utility and maximization of the surprisal, incorporating our state of knowledge. For example, Bordley and LiCalzi (2000), noted that the strategy that we should maximize our ignorance⁶ is equivalent to maximizing expected utility. Consequently, the expected utility model of Von Neumann and Morgenstern (1944) for preferences under uncertainty gave rise to a concept of utility-probability duality (Abbas and Matheson, 2004, Abbas, 2006, Castagnoli and LiCalzi, 1996). Jordaan (2005), for example, remarked that this duality is very much "like force and displacement in mechanics".

The duality arises because the "economic" agent, apart from having a preference over outcomes, simultaneously exhibits an attitude towards risk. Consequently, integrating equation 9.1 by parts allows the product to be specified in alternative forms. Berhold (1973), for example, rescales probability functions yielding reasonable utility functions while Castagnoli and LiCalzi (1996) normalize utility functions into probability functions. The certainty equivalence principle arising from the maximization of expected utility, thus, gives rise to a mirror concept of aspiration equivalence principle, often associated with Markowitz (1952). The scaling of the utility func-

⁶The rationale here is that the seat of our experience is within us. So, if we know nothing about the external, everything external is equiprobable. As we obtain some information about the system, the entropy represents, in some average sense, the number of states that are governed by $P(\cdot)$ so that the broader the distribution, the bigger the entropy. Consequently, maximizing ignorance should be observed to express an epistemically modest claim; i.e. the probability distribution should make the least claim beyond prior knowledge.

tion, for this equivalence, assumes a step utility function, also called an aspiration utility function, with the same expected utility as the nonscaled utility function.

9.1.2 The Aspiration utility

The aspiration utility function partitions the sample space into two, unsatisfactory and satisfactory; i.e. the utility function is an all-or-nothing function. If aspirations are satisfied, it provides a unit of happiness, and if not, it provides nothing. Briefly, an aspiration level, θ , is first defined. If the outcome is larger than θ , a util is received and if the outcome is lower than θ , zero units of util are received. The aspiration utility function can thus be written as

$$U(q) = \begin{cases} 1, & \text{if } q \geq \theta \\ 0, & \text{if } q < \theta \end{cases} \quad (9.2)$$

Representation of utility functions as weighted goals have been numerous (see for example Uckelman *et al* (2009)). In the context of dynamic choice situations, Simon (1955) suggested that binary goals could simplify decision problems a great deal. Such a scaling and subsequently reversing the roles of the utility function with the probability function has recently been done by Abbas (2006) who scaled the utility function between zero and unity so that it obeys the laws governing a cumulative probability function. Subsequently, *preference probabilities*⁷ allow any complex lottery to be equivalent to another lottery with only two outcomes, the best and the worst. So, defining $U(q)$ to be a normalized utility function, $f(q)$ be the probability density function and \tilde{q} be the aspiration equivalent of the lottery,

$$\begin{aligned} U(\tilde{q}) &\triangleq \text{Expected utility} = \int_{-\infty}^{\infty} U(q) dF(q) \\ &= \int_{-\infty}^{\infty} U(q) f(q) dq \end{aligned}$$

⁷Preference probability is the term used by Howard (1992).

$$= 1 - \int_{-\infty}^{\infty} F(q) dU(q) \quad (9.3)$$

So, under the normalization of $U(\cdot)$,

$$\text{Expected Utility} = 1 - \text{Expected Disutility} \quad (9.4)$$

9.2 Temporal aspiration

Definitions of aspiration from dictionaries, however, often involve an object of desire in the future and, accordingly, I seek to define the level of aspiration so as to incorporate time. To that end, I analyze the path structure of the quantity, $Q(t)$, representing utility at time t , by specifying the instantaneous aspiration utility, $\Delta_h Q(t) \equiv Q(t+h) - Q(t)$ in the limit that h goes to zero. I will make an important assumption for that purpose:

- **Assumption:** The rate at which we aspire for 1 util in a future is the same as the (negative) rate at which we discount 1 util from that future.

With this assumption, I seek a dynamic level, $\theta(t)$, that includes a delay between the moment that we aspire and the moment that we expect for our desires to be achieved, t . Defining a general discount weight function, $w(t)$, we have

$$\frac{\partial E(Q(t))/\partial t}{E(Q(t))} = -\frac{\partial w(t)/\partial t}{w(t)} \quad (9.5)$$

If we suppose that the utility level at time $t = T$ is at q , then for a small enough time interval, h , we have

$$E(Q(T+h) - Q(T) \mid Q(T) = q) = -qh \frac{w'(T)}{w(T)} \quad (9.6)$$

The sole purpose in considering units of the util is that we can consider a vast array of distribution functions at a specific time. Since,

$Q(t)$ is increasing, $Q(t) \leq Q(t + h)$ so that t provides an ordering of the random variables. If we fix a time, say $t = T$, then we can define a distribution function, $S(T) = F(Q(T))$, wherefore

$$\begin{aligned} P\left(S(T) \leq \frac{-qw'(T)}{w(T)}\right) &= P\left(F(Q(T)) \leq \frac{-qw'(T)}{w(T)}\right) \\ &= P\left(Q(T) \leq F^{-1}\left(\frac{-qw'(T)}{w(T)}\right)\right) \\ &= FF^{-1}\left(\frac{-qw'(T)}{w(T)}\right) \\ &= \frac{-qw'(T)}{w(T)} \end{aligned}$$

So, we can focus solely on probability distributions, without any utility functions. Consequently, assuming that now, time $t = 0$, the level of utility is at q^0 , and a similar constrained maximization as the previous chapter, we have the stochastic distribution:

$$P(q^0, q^t) = \frac{\Gamma(q^t)}{\Gamma(q^0 + 1)\Gamma(q^t - q^0)} w(t)^{q^0} (1 - w(t))^{q^t - q^0} \quad (9.7)$$

We thus have a temporal aspiration path in the mirror world of this duality, where we can represent utility functions as probability distributions. With util as measurement, from equation 9.2, we can identify that $\Delta_h Q(t) \equiv U(q)$ in the limit that $h \downarrow 0$ and from equations 9.4 and 9.6, we identify the *Expected inatantaneous utility gain* $\equiv q\left(\frac{-w'(t)}{w(t)}\right)$. So, keeping $w(t)$ and $Q(0) = q^0$ fixed, the possible future states increases with time, which is intuitive given our current state of knowledge since the further in time the event we try to predict, the more the uncertainty. Alternatively, if we observe the individual whithin a small interval, we can claim that some states are impossible to be realized since the weight $w(t)$ serves to "regulate", in some sense, the flow of surprisal.

At the present time, $t = 0$, our ignorance is null and we remark that q^0 , being given, has a probability of occurance equal to one. With

different starting points, $P(\cdot)$ admits different measures. So, we again fix a future time, say $t = T$ which will be our consideration. Ramsey's derivation, along with the system in chapter 7, both consider only two time points, $t = 0$ and $t = T$. Then, with given time points, $t = 0$ and $t = T$, considering the Euclidean plane spanned by the axes q^0 and q^T , $P(\cdot)$ traces a surface in the Euclidean octant with orthogonal projections onto the plane $\in (0, \infty)$ within the upper triangle $0 < q^0 \leq q^t$. Thus, viewing $P(q^T, q^0)$ as a parametrization of the random variables, we have a maximum surprisal map; i.e. a collection of maximum surprisal curves. Regardless of the surface traced, $P(\cdot)$ merely serves to indicate intertemporal indifference amounts that best reflects the current state of knowledge of an immortal society.

9.2.1 The marginal monetary value of health

We can now discuss the mirror concept of marginal value for society, through probabilistic arguments. In their first version of Expected Utility Theory, Von Neumann and Morgenstern (1944) also used a probabilistic indifference argument and assigned utility values ranging from zero to one. Similarly, the probabutility concept allows our analyses to be based solely in probabilistic formulations. Then, the rate of change of a future quantity in terms of it's current quantity is derived by computing the relative contribution of q^T compared to q^0 influencing $P(\cdot)$. The utility analogy of the latter computation is equation 3.3. In the dual world, we consider the differential in $P(\cdot)$, given by equation 9.7, at the point (q^0, q^T)

$$dP(q^0, q^T) = \frac{\partial P(q^0, q^T)}{\partial q^0} dq^0 + \frac{\partial P(q^0, q^T)}{\partial q^T} dq^T \quad (9.8)$$

Differentiating $P(\cdot)$ with respect to q^0 gives

$$\frac{\partial P(q^0, q^T)}{\partial q^0} = P(q^0, q^T) (\gamma - 2\text{ArTanh}(1 - 2w(T)) - H_{q^0} + \psi(q^T - q^0)) \quad (9.9)$$

where

γ is the Euler-Mascheroni constant

$\psi(\cdot)$ is the digamma function

and H_{q^0} is the q^0 -th harmonic number.

The partial differential with respect to q^T is

$$\frac{\partial P(q^0, q^T)}{\partial q^T} = P(q^0, q^T) (\psi(q^T) - \psi(q^T - q^0) + \ln(1 - w(t))) \quad (9.10)$$

Since, without moving off the probability curve $P(q^0, q^T)$, if one changes q^0 by dq^0 , then one must also change q^T by dq^T so that $P(\cdot)$ remains unchanged; from equation 9.8, we set $dP(\cdot) = 0$ and substitute therein equations 9.9 and 9.10 to get

$$\frac{dq^T}{dq^0} = \frac{\gamma - 2\text{ArTanh}(1 - 2w(T)) - H_{q^0} + \psi(q^T - q^0)}{\psi(q^T) - \psi(q^T - q^0) + \ln(1 - w(t))} \quad (9.11)$$

Let us now consider a society composed of only health and wealth, h and m , respectively. As theoretically argued in chapter 3 and as empirically observed in chapter 5, I will assume a discount rate for health to be lower than the *SDR* or the *srtP*, say $v(T)$. Then, as in chapter 7, where we let our social welfare function be summarised over money streams and health streams, $W(m^t, h^t)$, I now let the relative change of future money in terms of current money be denoted by

$$-\frac{\partial m^0}{\partial m^T} \Big|_{dP=0} = -\frac{\ln(1 - w(T)) + \psi(m^T) - \psi(m^T - m^0)}{\gamma - 2\text{ArTanh}(1 - 2w(T)) - H_{m^0} + \psi(m^T - m^0)}$$

and the relative change of future health in terms of current health be denoted by

$$-\frac{\partial h^0}{\partial h^T} \Big|_{dP=0} = -\frac{\ln(1 - v(T)) + \psi(h^T) - \psi(h^T - h^0)}{\gamma - 2\text{ArTanh}(1 - 2v(T)) - H_{h^0} + \psi(h^T - h^0)}$$

The (negative) social relative rate of change of money with respect to health at time t , defined as

$$-\frac{\partial m^t}{\partial h^t} \Big|_{dP=0} = v^t$$

then satisfies our theorem, in chapter 7, with $n = 2$:

$$\begin{bmatrix} (1 - v^T) & v^0 \\ v^T & (1 - v^0) \end{bmatrix} \begin{bmatrix} \frac{\partial h^T}{\partial h^0} \\ \frac{\partial m^T}{\partial m^0} \end{bmatrix} = \begin{bmatrix} \frac{\partial h^T}{\partial h^0} \\ \frac{\partial m^T}{\partial m^0} \end{bmatrix}$$

and has solution, analogous to equation 7.7:

$$\begin{aligned} v^T &= \left(\frac{\partial m^T / \partial m^0}{\partial h^T / \partial h^0} \right) v^0 \\ &= \frac{\left(\frac{\gamma - 2 \text{ArTanh}(1 - 2w(T)) - H_{m^0} + \psi(m^T - m^0)}{\ln(1 - w(T)) + \psi(m^T) - \psi(m^T - m^0)} \right)}{\left(\frac{\gamma - 2 \text{ArTanh}(1 - 2v(T)) - H_{h^0} + \psi(h^T - h^0)}{\ln(1 - v(T)) + \psi(h^T) - \psi(h^T - h^0)} \right)} v^0 \end{aligned} \quad (9.12)$$

Thus, we have a temporal-to-spacial mapping allowing us, with given indifference amounts for money and for health and a given current relative change in the monetary value of health, to investigate the timely relative monetary values. As a numerical example, consider the case of the Netherlands, with a 4% discount rate for money and a 1.5% discount rate for health. Assuming a current marginal rate substitution between money and health to be, say 30,000€/QALY. Then in one years time, the marginal rate of substitution, given current economic consensus, is approximately⁸ 30,000(1.025)€/QALY.

However, time preferences are dissimilar from intertemporal indifferencees where I shall take into account the "difference in the contributing powers" of q^0 and q^T to $P(\cdot)$, respectively. Subsequently, with the given current marginal substitution between money and health, I shall interpret the discount rates for money and for health as the aspiration rates. Then, on the Euclidean space (h^T, m^T) , equation 9.12 admits relative surprisal maps. At the point of our expectations, $m^T = 30000(1.04)$ € and $h^T = 1(1.015)$ QALYs, and we find that the growing relative value of health, is a lot more; approximately 30,000(1.065)€/QALY. The latter is not surprising since *surprisal* is maximized.

⁸1.025 = $\frac{1.04}{1.015}$

9.3 Discussion

This chapter makes a very important assumption; our aspiration rate is equivalent to our rate of time preference⁹. Under that assumption, it is feasible investigate consistency arguments regarding society's aspirations for the future with regards to several commodities, including health¹⁰. Divergences across different types of commodities, if any, could be solely represented through probability distributions, due to the probabutility concept. The notion of probability preference is closely related to our ignorance, measured by the surprisal.

It is interesting that, by definition, surprisal is not only a function of the properties of a system, but also, a function of our state of knowledge of that system (Note that outcomes are not solely states of nature). Consequently, scaling intertemporal utility or welfare functions and using the aspiration levels to trace a probabilistic path across time provides an alternative way to formulate decision rules across time. I should, however, mention, that regardless of future aspirations, ethical rates should always be preferred compared to the rate of time preference.

It seems that for such a case, we should encounter fewer problems, such as interpersonal comparison of utility or other issues argued in Elster and Roemer (1991), since we can solely observe surprisal maximization when occurrences in time are unknown¹¹. My proposition depends on future wants and their probabilities. However, one could devise rates for needs, first of all, and, only then, adjust those rates to allow for wants, for example. Luce and Raiffa (1957)

⁹This assumption requires further investigation since sequence studies often differ from time preference studies, as we have seen in chapter 4, and aspiration rate is novel.

¹⁰We already mentioned that we do not regard health as a commodity. However, in an informal survey, majority of respondents chose to maximize quality of life for posterity as opposed to maximizing consumption or income.

¹¹The rates of aspiration could be proxied solely by observed growth rates.

provide an elaborate literature for a one-person decision theory, although we all possess empathetic preferences that govern our ethical concerns (Hammond, 1976, Harsanyi, 1977).

9.a Appendix: An illustration on discounted life-expectancy

We have seen that the spacial characteristics might be derived from the temporal influences and vice-versa through the matrix-vector system provided in chapter 7 with the temporal distribution centered on the weighted future values as in chapter 8. In general, for temporal distributions, with an *external* given deterministic discount rate, it is known that the expected discounted outcome equal the discounted expectation, as a consequence of integration by parts. In this section, as an analogy, I provide an example for discounted life expectancy. That is, if a known cohort is observed, the life contingent probabilities provide an exact mapping onto the time-contingent demographical distributions.

I provide a basic proof that averaging of discounted human life years equals discounting of life expectancy¹². In practice, the present value of life years gained due to an intervention is evaluated by discounting at discrete time intervals. The interval, generally used, is traditionally yearly intervals. Health economists generally opt for three main methodologies, namely discounting life years from the beginning of the extra life years gained or from the end of year of each life years gained, or from the mid-year of the extra life year gained. This section shows that time averaging (i.e. when the probability density function is specified over time such as the case for life duration) of discounted life years and discounted life expectancy are equal.

As a general methodology, I shall treat life gained in this chapter as a continuous outcome with time with yearly units of measurements. With data about extra months of life gained, or extra weeks of life gained, or days gained, one might make the necessary adjustments. If months are considered, for example, then the outcome would be

¹²A similar result is well-known to actuaries involved in life insurances.

1/12 of a year; or if days are considered, then the outcome would be 1/365 of a year. Similarly, for an infinitesimal interval $(t, t + \Delta t)$, the life lived is Δt provided that the intervention extends the life through that interval.

Probability

Thus, there is a probability measure attached to living an extra instant, and consequently we will only discount the life of an individual for as long as she is alive. Let us consider the random variable, $T(x)$, the future lifetime of an individual who is aged x now, at time $t = 0$. To make probability statements about $T(x)$, I will use demographic notations

$${}_tq_x = P(T(x) \leq t)$$

And

$${}_tp_x = P(T(x) > t) = 1 - {}_tq_x$$

Where ${}_tq_x$ is the probability of dying within the next t years and ${}_tp_x$ is the probability that the individual will attain age $x + t$. The probability density function, p.d.f, of $T(x)$, $f_{T(x)}(t)$, can be found using traditional demographical methodologies. That is, the probability that the individual aged x dies exactly after t years is the probability that the individual survives to time t and dies within the period $(t, t + \Delta t)$; in the limit Δt tends to zero. Sufficient to specify the survival distribution is the force of mortality:

$$\begin{aligned} & P(t < T(x) \leq t + \Delta t | T(x) > t) \\ &= \frac{P(T(x) \leq t + \Delta t) - P(T(x) \leq t)}{1 - P(T(x) \leq t)} \\ &\cong \frac{\partial / \partial t P(T(x) \leq t) \Delta t}{1 - P(T(x) \leq t)} = \frac{f_{T(x)}(t) \Delta t}{1 - F_{T(x)}(t)} \end{aligned}$$

Where the function $\frac{f_{T(x)}(t)}{1-F_{T(x)}(t)}$ has a conditional probability density interpretation and denoted by $\mu(x+t)$. The p.d.f of $T(x)$, the probability that a life aged x survives to time t and dies instantly at that time is then given by

$$f_{T(x)}(t) = {}_t p_x \mu(x+t)$$

Parametric laws of mortality are numerous. Some examples of the probability that an individual aged x dies exactly after t years are:

1. **Constant force of Mortality** $\mu(x+t) = \mu$ and ${}_t p_x = e^{-\mu t}$
2. **De Moivre's Law** $\mu(x+t) = \frac{1}{w-x-t}$ with ${}_t p_x = \frac{w-x-t}{w-x}$
3. **Generalized De Moivre's Law** $\mu(x+t) = \frac{\alpha}{w-x-t}$ with ${}_t p_x = \left(\frac{w-x-t}{w-x}\right)^\alpha$

with the usual restrictions.

Discount weight

I shall assume, without loss of generality, that valuations are carried out in the present time. Let us assume, for now, the deterministic case of t years of life lived with certainty from now. We assume that an extra year of life is discounted at a rate of, say, $r\%$ per annum. Supposing we discount from, say, the end of each life year gained, we have a present value of years of life lived, given by $\sum_{i=1}^t \left(\frac{1}{1+r}\right)^i$. Letting the discount factor per year be $w(t) = w \forall t$, we have

$$\sum_{i=1}^t \left(\frac{1}{1+r}\right)^i = \sum_{i=1}^t w^i = \frac{1-w^{t+1}}{1-w}$$

In the case of a continuous lifetime, we first of all need to find an infinitesimal discount rate. A single life year gained at time $t = T$ has a present worth of w^T . Denoting life years gained at time T by $L(T)$ and the continuous discount rate by δ , we have

$$L(0) = L(T) w^T$$

$$\begin{aligned}
&\rightarrow L(T) = L(0)(1+r)^T \\
&= L(0) \lim_{\Delta T \rightarrow 0} (1 + \delta \Delta T)^{\frac{T}{\Delta T}} \\
&= L(0)e^{\delta T}
\end{aligned}$$

Thus the infinitesimal continuous discount factor is $w^t = e^{-\delta t}$. Plugging this discount factor for the present worth of T years of life lived continuously and with certainty, we have $\frac{1-w^t}{\delta}$.

Equality of Averaging discounted life years and Discounting life expectancy

However, we have a probability measure attached with the survival of a life. Consider the random variable Y , the total present discounted life years lived continuously. That is

$$Y = \frac{1 - (1+r)^{-t}}{\delta} = \frac{1 - w^t}{\delta}$$

Then the expected value of the total discounted life years, the expected value of Y , $E(Y) = \int_0^\infty \frac{1-w^t}{\delta} {}_t p_x \mu(x+t) dt$. But, using integration by parts,

$$\begin{aligned}
E(Y) &= -\frac{1-w^t}{\delta} {}_t p_x \Big|_0^\infty + \int_0^\infty e^{-\delta t} {}_t p_x dt \\
&= 0 + \int_0^\infty e^{-\delta t} {}_t p_x dt \\
&= \int_0^\infty e^{-\delta t} {}_t p_x dt
\end{aligned}$$

which is the net present worth obtained by discounting the function of time representing the expected life years, $\int_0^\infty w^t {}_t p_x dt$. In such a case, it appears irrelevant which order we chose to carry out our analysis; averaging or discounting first. We recall that we used the p.d.f $f_{T(x)}(t)$ which is ${}_t p_x \mu(x+t)$. However, ${}_t p_x$, the probability that a life aged x will attain age $x+t$ is defined as the number of

people alive, at age $x + t$ relative to an original cohort, l_0 newborn, divided by the number of people alive, relative to the same cohort, l_0 newborn, at age x . Similarly, the force of mortality is also totally defined in terms of the demographic notations. Hence, the validity of this section with respect to chapter 7 rests upon whether the p.d.f $f_{T(x)}(t) = {}_t p_x \mu(x + t)$ maps the current population demography and a future time t population demography.

Example: Let us take an example of a population with a constant force of mortality, $\mu(x + t) = 0.005$ and a continuously compounded discount rate of 1.5% for a life year. Then, since ${}_t p_x = e^{-\mu t}$, we have

$$\begin{aligned} \int_0^\infty \frac{1 - w^t}{\delta} {}_t p_x \mu dt &= \int_0^\infty \frac{1 - e^{-0.015t}}{0.015} e^{-0.005t} (0.005) dt = 50 \\ &\equiv \int_0^\infty w^t {}_t p_x dt = \int_0^\infty e^{-0.015t} e^{-0.005t} dt = 50 \end{aligned}$$

I would like to mention, however, that intragenerational or intralife-time discounting is not a practice that ethical experts would advocate. One might argue that a baby's life is a life, just like an adult's life is a life. Discounting the remaining life expectancy of a life aged x violates the core principles of justice. This appendix, however, serves to illustrate a mathematical fact that discounting an average future life, under a temporal distribution, equals averaging the function of time representing the discounted future life, under that same temporal distribution. Consequently, I shall limit the discussion of this appendix minimal.

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Chapter 10

General Discussion

While this thesis argues the need for differential discounting and proposes a framework for that matter, I fail to provide a theoretically correct health-specific discount rate. However, in order to remain within ethical realms, a lower discount rate for health compared to cost is strongly motivated for the purpose of cost-effectiveness analyses; a zero discount rate being also a realistic possibility. Yet another example is the infant pneumococcal vaccination programs that have been implemented in the past 10 years worldwide, aiming primarily to avert invasive pneumococcal disease in infants and related mortality. Saving an infant's life might involve 80 life years gained (approximate average life expectancy at birth) from the clinician's point of view (Rozenbaum *et al*, 2011). From the health economist's point of view, however, applying the broadly consented *SDR* to the stream of life years reduces the 80 physical life years to approximately 20 discounted life years (at 4% discount rate as an example). Again, this 4-fold reduction in the life years gained would grossly be mirrored by a 4-fold increase in the cost-effectiveness ratio of pneumococcal vaccination (Hubben *et al*, 2007, Postma, 2008).

Equal discounting is primarily supported by pragmatic and historic motivations, rather than sound evidence and reasoning. The prescription of an equal rate, motivated by the postponement paradox

of Keeler and Cretin (1983), has been contended with a single argument; *The monetary value of health is not necessarily a constant in time* (Gravelle and Smith, 2001). Fortunately, the expertise in the area of discounting in health economics is rapidly accumulating, involving thorough discussions, complex analyses and strengthening alignment with empirical situations. This expertise, more and more, comprises the specificities of a human life in discounting rules, enhancing fair and valid discount rates and, albeit, fair and valid assessments of costs and benefits with different timings, for both public health and pharmaceutical strategies. Currently, the Netherlands and Belgium prescribe a lower discount rate for health gains as opposed to the rate for costs and savings (CvZ , 2013, KCE , 2013).

With a broad consensus to discount (CvZ , 2013, ISPOR, 2013, KCE , 2013), the specific source for inferring and pooling the exact discount rate for health, however, remains obscure to us. Is it directly related to the real rate of return on investment, the interest rate on government bonds or the long-term rate of economic growth (Keeler and Cretin, 1983) or on growth rate of life expectancy or should the discount rate, theoretically, be based on the time-preference for health (Olsen, 1993)? Various countries, amongst which the Netherlands (CvZ , 2013), have specified in their guidelines that, preferably, population preferences ought to be taken; i.e. costs, savings, life years and quality of life should be discounted at the rate of time preference; irrespective of whom the benefits fall onto and who “suffers”. Empirical evidences in discounting typically indicate decreasing discount rates such as hyperbolic discounting (Westra *et al*, 2012). The latter has been taken into consideration by several authors; i.e. it has been suggested to apply lower discount rates the further in the future the outcome. The UK treasury (2013) as well as Bazelon (2002) and Beutels *et al* (2008) have suggested specific numericals for that matter.

With regards to discounting health outcomes, this could imply that, rather than deriving rates and relations or inferences from rates on

government bonds, the rates should be directly related to the time preferences of the population concerned. With our empirical results, the annual discount rate for health consistently lies below the annual discount rate of money, with that of health being at least half that of money for short term delays and this factor decreases to at least one eighth with increasing time of delay. One might suppose that, for example, with the Dutch 4% rate for money, the discount rate to be applied for health would have the same relative decrease¹. Such attempts to near ethics, however, require theoretical underpinnings.

Ratification for an economy supporting differential discounting with non-constant rates appeared monumental. In order to admit ground-works in economic theory for differential discounting, I relax the assumption of a consumption-invariant discount rate allowing, also, for rates that are not time-invariant. I form a model-consistent expectation of an n -commodity economy where each commodity grows according to their n different growth functions. Using the variability in the specious present (Valera, 1999), coupled with the expectation of future consumption given current one, I assign probability measures by maximising the *surprisal*. I obtain the probability that an animal/individual/society forsaking a certain quantity now, will be indifferent to some quantity at a future time. Although appropriateness criteria for questionnaire investigations are not met, the distribution that I propose might, rightfully, investigate the time until a compensation is demanded; such as the famous Stanford marshmallow experiment (Mischel *et al*, 1972).

Unfortunately, I remain unsuccessful in devising a health-specific discount rate. However, in like manner to the calibration of the Ramsey (1928) discount rate by setting the objective to be attaining *Bliss*, calibration of a health-specific discount rate would entail formulating an adequate, well reasoned, and ethically accurate government

¹Recalling that the third aim of chapter 5 was to elicit time preference for money, this elicitation is mapped onto the 4% so that the differential rate for health is devised.

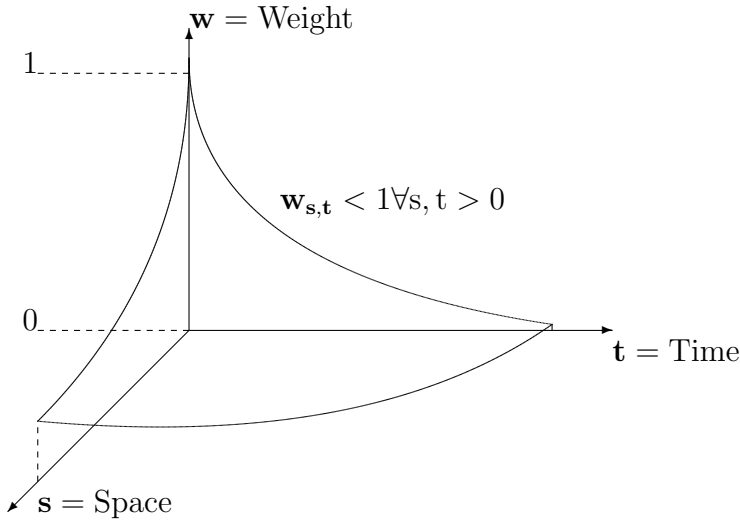
objective. Vaccination benefits often extend beyond one's current generation. Hence, assuming the argument of Diamond (1967) that social choices, require more information than the total expected utility, as for individual choices, but also information about the distribution of welfare among society², I suggest considerations of intergenerational utilitarianism. Accordingly, from ethical perspectives, one could focus solely on surprisal maximization, given current state of knowledge and expected aspirations.

Oppositions to the use of individual's own discount rates have been abundant; Pigou (1920), for example, argued: "*The state should protect the interests of the future in some degree against the effects of our irrational discounting and of our preference for ourselves over our descendants. It is the clear duty of Government, which is the trustee for unborn generations as well as for its present citizens, to watch over, and if need be, by legislative enactment, to defend the exhaustible natural resources of the country from rash and reckless spoliation.*" Since discounting of human lives drastically affects economic evaluations such as cost-effectiveness of vaccinations and subsequently future generation's health, we could see current generation as having a monopoly over the distribution of health across time. As Sen (1961) pointed out, if democracy means that all the people affected by a decision must themselves make the decision, then there can be no democratic solution to the intergenerational problem, future generations being unborn yet.

An even more challenging task remains formulation of a health-specific discount rate with the utopian goal of global welfare optimality. The considerations that one ought to take, to that end, are much more complex. Global welfare maximization requires that rich countries spend more on foreign lives than on national lives considering the higher number of individuals in poorer countries than in richer ones. However, the opposite is observed. Strong evidences

²Society here means *intertemporal society*.

were provided by economists(see, for example, Poulos and Whittington (2000)) that rich countries do discount lives of poorer countries (see Figure below).



Graphical presentation of the implied discount rate where the axis, w , is the weight attached, the axis, s , being location in space, and the axis, t , is time. The origin, represents self now or national now. The hypothetical curve on the Euclidean quadrant spanned by the axes w/s might thus represent our discount factor from self to others or individual to others and might also be representative of own country of residence to other lower-income countries, for example. The axis, t , is representative of time and the curve on the quadrant w/t represents the discount factor of future outcomes for an individual or self, or national. The space spanned by the 3-dimensional Euclidean orthant gives some indications of the discount factor one attaches to the preference of others through time.

Figure 10.1: Space-Time discounting

Although spacial discounting is beyond the purpose of this thesis, the derivation of a health-specific discount rate intuitively appears

to rest upon non-classical utilitarianism. I briefly mentioned the Rawlsian maxi-min principle, in chapter 7, which is also consistent with Brown-Weiss (1989) approach. The maxi-min principle is the strongest in assuring that the least fortunate of future generation's levels of consumption are at least as great as those of the least fortunate of the current generation (Bruce *et al*, 1995). With regards to health, in the absense of technical change, consumption per head is then ensured to be similar for all future generations. Rawls (1972) strategy, however, though popular in game theory³, does not build on a unanimous axiomatic framework; much work remains to be done here, especially in formulating an economic theory with clear distinctions between consumer and consumee.

The distribution of health across time being a consequence of our current policies, is thus very much dependent on the choice of discount rate for health. Assuming a certain allocation of funds for certain health interventions, labour markets, which are highly correlated to health, are also altered; whether regarding the average age of the population work force or the number of individuals in the population. Numerous empirical investigations have shown that health states have impacts on employment, wages and hours worked, as well as GDP (Bhargava *et al*, 2001, Pelkowski and Berger, 2004). Consequently, from an ethical view point, for a country-specific discount rate for health, the maximization problem of the government should not risk the survival of their sovereignty. Devising well-reasoned aspirations, in particular, devising sustainable rules, provide numerous challenges. One such consideration is global debt. Sovereign debt defaults have been numerous since creditors do not have a clearly defined claim on the sovereign's assets (Reinhart and Rogoff, 2008). Debts that have been hidden for several years are, however, gradually being unraveled. Council Regulation of the European Union number (479) recently required EU Member Countries to publish their debt information in an explicit manner. In to-

³The maxi-min solution equals the Nash equilibrium in zero-sum games.

day's modern world, it should be noted that if global debts were to be repaid, not a cent would be in circulation (IMF, 2013). Furthermore, the IMF (2013) also predicts that when baby boomers come to retire, global working population will be within a smaller work force percentage than it is now, for many more years to come (IMF, 2013). Ergo, still further work on the topic is required.

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Summary

This thesis first points out the ethical debates around *the devaluation* of future health with respect to cost-effectiveness of vaccination programs¹. I argue the need to bring ethics in line with guidelines. Briefly describing how a CEA is carried out and pointing out the bias for cures to be cost-effective compared to preventions, my colleagues and I list some attempts to achieve morally correct CEAs. However, ethically correct decision rules should also be theoretically correct². Theory suggests adoption of the Ramsey discount rate. I derive the rate and discuss its appropriateness for costs. However, I provide theoretical arguments that, firstly, the pure rate is circular reasoning for human lives; secondly, the growth rate should be based on growth in life expectancy and, thirdly, the elasticity for a life year is assumed to derive from a linear utility function for life years. I also provide a list of growth rates of GDP compared to life expectancy, all of which suggest moderate growth in life expectancy compared to growth in GDP³.

Given the lack of motivation for an equal rate, my colleagues and I sought to investigate whether individuals discount health at a lower rate, so we designed a questionnaire that would allow us to compare discount rates for health compared to money, while keeping all questions, as far as possible, similar for both money and health⁴.

¹Chapter 1

²Chapter 2

³Chapter 3

⁴Chapter 4

Next we carried out our empirical investigation on a representative sample of the Dutch population. We find differential rates, as well as differential rates among population characteristics such as health states⁵.

Given the lack of theoretical motivation for equal discounting and our empirical results, differential discounting is strongly supported. Majority of bodies however suggest an equal rate, primarily based on the Keeler and Cretin paradox. The paradox derives from a well-known general economic paradox. Koopmans labelled it *the paradox of the indefinitely postponed splurge*⁶. Not discounting benefits at the social rate of time preference would cause distant generations to be *utility monsters*, due to excessive sacrifice of the current generation. However, a single argument allowing for differential rate for health is the fact that the monetary value of health need not be constant in time⁶. Given the need for differential discounting, we form a model-consistent expectation of an n -commodity economy where each commodity grows according to their n different growth functions⁷. Next, I add probability measures to each constituent of the system by assuming well-known psychological posits, namely the variability of the specious present⁸. Following that, I discuss the marriage between the deterministic system and the probability measures and propose an aspiration rate⁹.

Next, I discuss that although I argued the need and attempt to provide a framework for differential discounting, I fail to investigate the exact source of pooling a health-specific rate. I suggest that the calibration of a health-specific discount rate would entail formulating an adequate, well-reasoned, and ethically accurate government objective. I argue that the primordial duty of the government should not

⁵Chapter 5

⁶Chapter 6

⁷Chapter 7

⁸Chapter 8

⁹Chapter 9

risk the survival of their sovereignty. Lastly, I discuss the Utopian goal of global welfare such as how health states affect discount rates, as we observed in chapter 5, and how that affects labour markets, and GDP, etc¹⁰.

¹⁰Chapter 10

Samenvatting

Dit proefschrift beschrijft eerst de ethische aspecten rondom het devalueren van gezondheidswinst in de toekomst bij kosten-effectiviteitsanalyse (KEA), met name bij vaccineren (disconteren). Ik beargumenteer dat richtlijnen over disconteren in lijn moeten zijn/komen met ethische argumenten. Na kort stil te staan bij de vertekening door disconteren in KEAs bij curatieve interventies t.o.v. preventieve, geef ik opties die in de literatuur zijn gegeven ethische KEAs te verichten met juiste disconteringseffecten. Echter, ethisch correcte beslisregels moeten ook theoretisch correct zijn, waarbij de theorie vooral Ramsey's discontering suggereert. Ik leid de corresponderende disconteringsvoet af en bediscussieer de bruikbaarheid voor kosten en besparingen, twee van de vier bestanddelen van de KEA. Echter, voor levensjaren (en kwaliteit van leven) is de bruikbaarheid beperkt: (i) cirkelredenering dreigt, (ii) de disconteringsvoet zou hier gerelateerd moeten zijn aan de groei in levensverwachting en (iii) aan de elasticiteit bij levensjaren in de Ramsey functie ligt een lineaire nutsfunctie ten grondslag. Met name, geef ik een lijst met groeivoeten in BNPs en in levensverwachtingen, die telkens suggereren dat de groei in de laatste aanzienlijk lager is dan in het BNP.

Gegeven het gebrek aan onderbouwing voor een gelijke disconteringsvoet voor kosten en effecten, zocht ik naar bewijs voor een andere (naar verwachting lagere) voet voor gezondheid dan voor geld. Met name, werd een vragenlijst ontwikkeld om dit bij personen te meten, met specifieke maar vergelijkbare vragen voor gezondheid

en geld. Deze vragenlijst werd - na een pilot bij farmacie studenten - vervolgens toegepast op een representatieve steekproef uit de Nederlandse bevolking. Daarbij werden verschillen aangetroffen tussen geld en gezondheid, maar ook bijvoorbeeld indien gestratificeerd naar gezondheidsstadium of indien bekeken in de tijd.

Op basis van theorie en (tevens bovenbeschreven) empirie wordt differentieel disconteren sterk ondersteund. De overgrote meerderheid van landen, instituten en richtlijnen suggereren echter nog steeds gelijke disconteringsvoeten voor geld en gezondheid, vooral onder verwijzing naar de zgn. "Keeler & Cretin paradox". Deze paradox is gerelateerd aan een bekende algemene paradox in de economie, i.e. "the paradox of the indefinitely postponed splurge" conform Koopmans. Hierbij wordt aangegeven dat het niet disconteren van winsten in de toekomst tegen de maatschappelijke disconteringsvoet het nut toekomstige generaties sterk bevoordeelt (utility monsters) tegen grote opofferingen van huidige generaties. Daar kan tegenin gebracht worden dat differentieel disconteren te verdedigen is vanuit een optiek dat de geldelijke waarde van gezondheid mogelijk niet constant is. Uitgaande dan toch van een noodzaak differentieel te disconteren, ontwikkelde ik een consistent model voor een n-goederen economie waarbij elk goed een specifieke groeifunctie heeft (bijvoorbeeld: kosten, besparingen, levensjaren en kwaliteit van leven). Vervolgens voeg ik probabilistische maten toe conform gangbare psychologische theorien. In een vervolghoofdstuk integreer de deterministische en de probabilistische aanpakken.

Vervolgens beschrijf ik het resterende probleem dat bij een noodzaak differentieel te disconteren, de exacte bron(nen) om de voet(en) voor gezondheidswinsten (levensjaren en kwaliteit) vooralsnog niet helder aan te geven is (zijn). Deze disconteringsvoet(en) zouden direct moeten aansluiten bij goed onderbouwde, specifiek gekozen en ethisch verantwoorde doelen van regeringsbeleid. Tenslotte bespreek ik de overwegingen in relatie tot het Utopische doel van wereldwijde welvaart; bijvoorbeeld, welke effecten hebben verschillen in gezondhei-

dszorgsystemen op disconteringsvoeten en vice versa hoe beïnvloedt discontering arbeidsmarkten, BNP etc.

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hope you stay out of trouble. Alette, I wish you success with your work. Michelle, it's sad you left to Rotterdam. Diana, you are very sweet and I like your hair colour. Pieter, your village stories are fun. Yunyu, sorry I couldn't help with accomodation in Berlin. Fanny, thank you for the epidemiological links. Nynke, we always meet in the elevator. Evelyn, you are always calm and sweet. Kim, I always enjoyed our conversations; you are very lively. Anja, you are a wonderful singer. Wanda, I always see you wearing your smile. Rolf, much thanks for the compliments about the painting; I am making you one. Anton, I am trying to make my hat myself; I will show you afterwards. Niesko, the summer is beginning to last more than two days per year. David and Abdel from the Mathematics department of Essex University, thank you so much for the interest that you showed in my research. Durrani, I still need to come visit the little princess and the littler prince. Rolando, spiderman cannot do half of what you can; Michael is proud of you. Maria, I am very happy my path came upon yours. Henry, I can never repay you for your advise, mate. Peter, the scotsman who liked to imagine he was chinese, I wish you good health, my friend. Farida, everytime we meet, our conversation turn philosophical. Jaya, sorry I missed your husband when he visited. Najia, many congratulations on the wedding; such a pity that I missed it. Alvin, if I am Tarzan, then you are Robin Hood. I feel priviledged that I encountered such great people. I am also grateful to GSK Bio (Wavre, Belgium) for providing the grant for this work.

As I am sitting in the office, writing this acknowledgement, I realize the trousers that I am wearing, posted to me by air mail, sewed by my tailor, from the cloth that the weaver weaved, from the bud that the cotton tree budded, from the billions of organisms that contributed solely to getting my body clothed; body that I accumulated from the pieces of this world in terms of food that I have eaten during the days I have lived. I am overwhelmed with gratitude. I am grateful to Providence.



The fact that we devalue the lives of our descendants has raised abundant ethical debates during the past few decades. Economists are saying that, because we value the present more than the future, this should not exclude anything; not even human lives. In an attempt to bring ethics in symphony with economics, this thesis proposes that our decisions depend on what we aspire for.

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